TRAINING MANUAL

IN

TOPOGRAPHY, MAP READING AND RECONNAISSANCE

Prepared by
MAJOR GEORGE R. SPALDING
CORPS OF ENGINEERS

Under the direction of the Chief of Engineers, U. S. Army

WASHINGTON
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TRAINING MANUAL IN TOPOGRAPHY, MAP READING, AND RECONNAISSANCE.

LESSON I.

TOPOGRAPHY.

1. So important is the influence of "*the lay of the land*" upon *military operations* that every officer must thoroughly acquaint himself at the earliest practicable moment with the topography of the area within which his responsibilities lie.

2. A mere knowledge of the main roads, of the direction and distances to near-by towns is by no means sufficient. He must know where the plains are, where the hills rise, how the streams run. *He must have a mental picture, a bird's-eye* view, impressed on his brain of the main *physical and military features of his territory.*

EROSION.

3. The faculty for grasping the topography of an area as a whole is rarely a natural one. "*To the beginner*" in the study of topography "*every hill is an isolated feature—the elevations and depressions of ground present to him nothing but irregularity and confusion.*"

4. As a matter of fact, there is a considerable degree of regularity and system in our present-day ground forms. Whatever may have been the irregularity and confusion of the original surface of the earth, practically all of the *ground forms which we know to-day are the result of erosion.*

5. It has been estimated that the average elevation of the land surface has been reduced 7,000 feet by erosion. Winds and waves, glaciers, chemical action, frost, and plant growth have all had their part in the breaking up and wearing down of the original rocks, but by far the larger portion of the *ground forms which exist to-day* have been carved out of the older formations by the *action of running water.*

MASTER LINES OF TOPOGRAPHY.

6. Not all the water in heavy rains can seep into the soil; much of it runs off. At one point a little stream begins; as it flows downward another joins it; soon several unite into a fair-sized creek, which rushes along carrying its burden of soil into the main valley, whence the river may carry it on toward the sea. This simple process continued for ages has sufficed *to carve out our deepest valleys.* It is continuing to deepen them year by year, and more and more to wear down the ridges between the valleys.

7. If our remaining ground forms are the result of such a process, it is manifest that the **drainage lines** of an area, together with the **ridge lines,** form a system of **master lines** which, once traced out and studied, will give a grasp of the main features of the topography that can be had in no other way.

CONTOURS.

8. The main features of the topography, however, while of first importance, are not sufficient for modern military operations. The military leader must also know the relative sizes of the valleys, the relative heights of the ridges, where the steep slopes are, where the hills are flat. This information may be indicated upon a map in several ways, but the best way is by a system of *contour lines.*

9. The beginner always has trouble with contours. He has learned perhaps that a contour is a line connecting points of equal elevation; that contours do not cross; that a contour line either closes on itself or both ends of it go off the map. But rules such as these, important as they may be in checking up a sketcher's work, do not help the beginner much. He is still hazy as to just what contours are and how they represent ground forms.

10. *It is true* that a contour is a line joining points of equal elevation. But such a definition is not precise. A *contour line is a line each point of which* has the *same elevation.* If one walks along a *contour line,* he *neither goes up nor down hill, but always on a level.* The surface of a quiet pond which has no outlet is practically level. Therefore its *shore line* is a *contour line.* If one follows it in the *direction of the hands of a clock* he will find that he must turn to the *left at every entering valley, walk up the valley* until he heads the water line, *cross the valley,* then turn again to the right, *following down the other side of the valley* to get around the *point of the hill or spur* which lies between it and the next valley. His course *will bend* to the left at every little inflowing *drainage line, cross it,* and *turn again* on the other side to *avoid leaving the level.* We see then that *valley contours apparently go in pairs,* that is, there is always one of the *same elevation* on each side of the *valley.* They form a sort of a V which opens out in the direction of water flow; point of the V upstream. Similarly the *spur contours* apparently *go in pairs.* They form a sort of U which opens out to the higher ground up the spur. (In glacial country the valleys are more nearly U shaped and the ridges sharp.)

11. **The typical contour,** then, is a curving line, alternately salient and reentrant, a series of rude V's for the valleys and U's for the spurs, the point of the V at the stream crossing, the curve of the U at the ridge crossing. If the extreme points of the V's and U's are determined and these points are connected by a curving line, opening out gradually as we go downstream from the points of the V and rounding out to the curve of the U, we get a line which will roughly represent the contour of the ground for that elevation. In other words, **the points of the V's, or the points where the contour crosses the stream, and the points of the U's, or the points where the contour crosses the line of the watershed between two valleys, are essential control points for the drawing of any contour line.**

12. The *single contour* which marked the shore line of the pond will, of course, give no indication of the shape of the hills which surround the pond. To *indicate these ground forms,* we must have a *number of contours,* each one of which shows the path which a man would follow if he walked around the hill forms on a level line. These level lines should be spaced at *equal intervals* in a *vertical direction.* In other words, if we take the shore-line contour as the datum contour, the next line should show the path which a man would follow if walking on a level line, say, 10 feet higher than the pond, and the

next, 20 feet higher, etc. This 10-foot *vertical interval* between the level lines is called the *contour interval* or vertical interval. It may be 1 foot, 5 feet, 10 feet, or 100 feet, or any other number of feet.

NOTE.—Our Field Service Regulations provide that the contour interval should be 10 feet for a military sketch made on a scale of 6 inches to the mile, 20 feet for a sketch on a scale of 3 inches to the mile, and, *in general,* the proper contour interval for any scale is to be found by dividing 60 by the number of inches on the scale which represents a mile on the ground. This system of scales is known as the United States Army Normal System. The British Army have a similar system, using 50 as the base instead of 60. In small-scale mapping these systems are usually not adhered to. The United States Geological Survey use an interval of 20 feet for their 1/62500 (about an inch to the mile) sheets, reducing this to 10 or even 5 feet in very flat country and increasing it for mountainous country.

CRITICAL POINTS.

13. It is manifestly impracticable in mapping to locate with instruments in the field the points where *each contour* crosses *each drainage line,* nor is this necessary. **No map can show every change of form of the ground.** We must satisfy ourselves, therefore, with locating the *critical points* of the *master lines* of the *ground.* Such points are the *heads,* the *changes in direction* and the *changes in slope* of the *drainage lines;* the *tops,* the *changes in direction,* and the *changes in slope* of the *ridge lines.* It will also be impracticable to determine the elevation of the head of every little gully or the location and elevation of every change in direction of *every drainage line* or of *every ridge line.* But we *must* locate the *master critical points* of the *master ridge* and *stream lines,* and the *elevation* of these points must be determined as well as the *location* of them. It is very important, however, to *plat* in every *gully,* though the elevation of its head be not determined, as these small drainage lines effect the shapes of the hill forms very *decidedly* and, furthermore, they are *landmarks* of *great value.* When the *critical points* and the *drainage net* has been located as above described, one who knows something of the laws of nature and remembers what a contour is can *interpolate* between the *master critical points* enough of the others to enable him to draw in all the contours.

INTERPOLATION.

14. Figure I is a skeleton sketch, giving the complete *drainage* net and a few of the *master critical points* of a section of the Fort Leavenworth (Kans.) Military Reservation. The main stream line of the area is shown as a solid line. The direction of flow is indicated by the arrow near elevation 790 (upper right-hand corner). The elevations of two points on the main stream are given (790 and 850). Smaller tributary drainage lines are indicated in dashed line. Elevations not on stream lines are *critical points* on the ridge lines.

15. *The problem* is to draw logical contours at 10 feet vertical intervals, with no other data than that given. The *first step* is to locate the points where desired contours cross the main stream line. As there are no falls between elevation 790 and elevation 850, it is *assumed* that the slope of the stream between these points is *nearly uniform,* becoming, however, a little *steeper* as the stream is *ascended.* Under this assumption the crossing points of contours 800, 810, 820, 830, and 840 are at once interpolated by eye on the main stream between 790 and 850. By interpolation between these the eleav-

tions of the points where each of the tributary ravines enter the main stream are secured. Between *these latter points* and the *heads of the ravines*, the elevations of which are given, the points where the contours cross each of the ravines are marked.

16. Each point shown along the *ridge lines* was chosen as being a critical point, that is, a point where the slope or direction of the ridge changes. From point to point along the ridge lines, it is assumed that the slope is uniform. The points where the intervening contours cross the ridge lines have, therefore, been interpolated. The result of interpolation along stream and ridge lines is shown in Figure II. As a rule the ridges and spurs of eroded hills point in the general

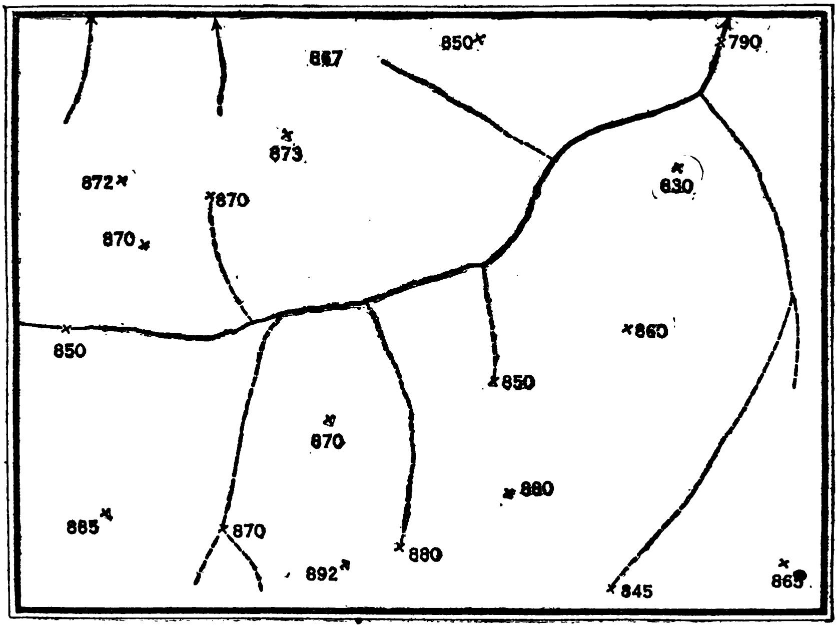


FIGURE I

direction of the junctions of the streams which have eroded them. Therefore, interpolations have been made in these directions.

DRAWING THE CONTOURS.

17. In sketching in open country, where all the features can be plainly seen, the method of drawing in the contours from this control is to *revisit* the *predetermined critical points* and to draw in all the contours which are needed to represent *one feature*, that is, one *spur or hill* at a time, using the drainage lines as the limits of the unit *feature* to be drawn. The smaller irregularities of the ground are brought out by slight changes in contour spacing and contour direction, here and there. (See Fig. III.) Figure IV is the completed sketch. It is very good representation of the ground.

PRACTICAL WORK.

18. At this point in his course, the beginner will be little benefited by practice with sketching instruments, or by going out and merely looking at the ground. The instructor should, if practicable, have several countors taped out on the ground with the particular purpose of showing the class how the contours run up the valleys and around the spurs. He will also require each member of the class to trace Figure I and, by following the text without reference to the other figures, to interpolate the contours on his tracing. The instructor will point out errors in method; particularly will he see to it that

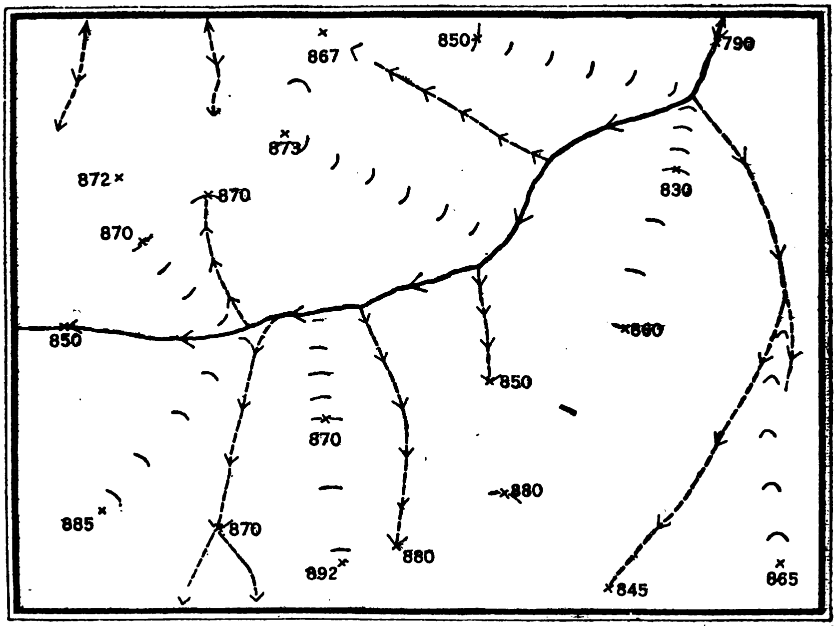


FIGURE II

interpolation be made *along* the *main* stream lines *first* and then along the tributary streams and that interpolation is *along ridge lines*, as the *ridges point.*

LESSON II.

MISTAKES IN DRAWING CONTOURS.

19. The characteristics of contours most frequently violated are listed in paragraph 20. These characteristics *must* be studied in the *light* of *what has been said* in Lesson I. There is no particular purpose in one's committing these *rules* to memory. What is necessary is that the beginner learn the reasons why contours have these characteristics. On Figure V notation of contour characteristics are noted by numbers, which refer to subnumbers of paragraph 20. It will be noted that the main stream is shown much steeper in its lower reaches than upper. This *might* be so, but is unusual; therefore should not be

so shown unless indicated by data on the skeleton. Contours marked *6* should turn up the stream lines. Contour marked *1* crosses same stream *twice*. It can not, therefore, be a level line, as the stream has *some* fall. What is *3*, a summit or a depression? Contours marked *11* follow no system whatever; ground shaped by water erosion will not be thus. The one marked *11a* (if it be a contour) indicates that one can walk directly across country from one stream to another on a level line. Would the streams be here if this were possible? Imagine yourself walking on contour marked *12;* will you always have lower ground on the same side?

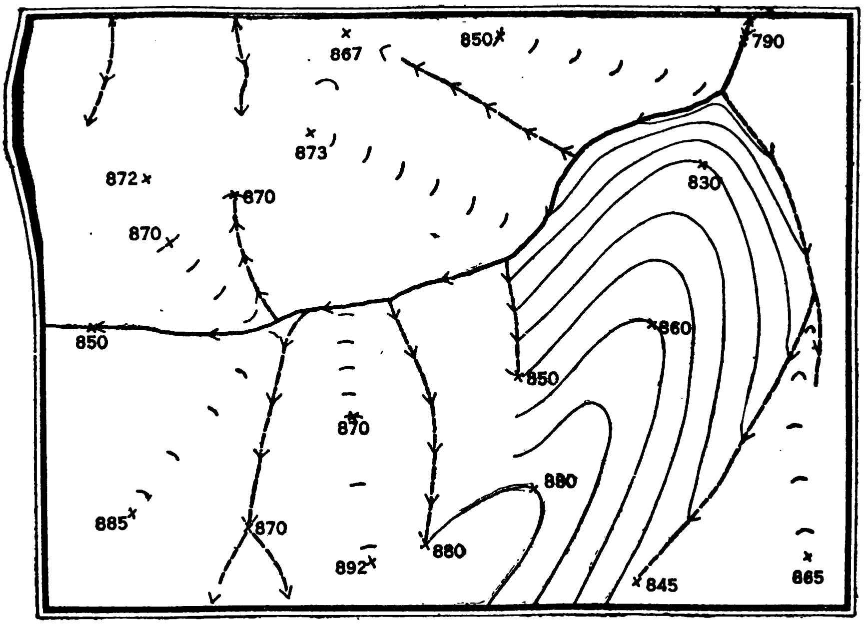


FIGURE III

CHARACTERISTICS OF CONTOURS.

20. (1) All points on any one *contour* have the *same elevation. It is a level line.*

(2) *Every contour closes* on itself, either within or beyond the limits of the map. In the latter case the ends of the contour line *will run to the edge of the map.*

(3) A contour which closes within the limits of the map indicates either a *summit* or a *depression.* In a depression there will usually be found a pond or a lake. If there is no water, the depression must be indicated in some way to differentiate it from a summit. A usual method is to *hachure the inner* side of the *depression contour.*

(4) *Contours never* split and never cross each other except in the case of an *overhanging cliff*, in which case there must be *two distinct intersections.* These cases are not common.

(5) Contours are spaced equally to represent a *uniform slope.* If the slope is a *plane surface* (i. e., if it has no irregularities due to erosion or other cause) the contours are *parallel* and *straight.*

(6) In *crossing a valley* the *contours* run up the valley on one side turning at the stream run back on the other side. In crossing a ridge the contours run to the ridge line and, turning, run back on the other side of the ridge.

(7) *Contours* are always at *right angles* to the lines of steepest slope. They, therefore, *cross the stream lines* and the *ridge lines* at *right angles.*

(8) The *contours* are *farther apart* at the *top* and *bottom* of an *eroded hill* than near the middle, because in these portions the slope is somewhat flatter.

(9) *Contours* are usually *closer together* near the *sources* of *streams*—as a stream is usually steeper near its source. This is not always so.

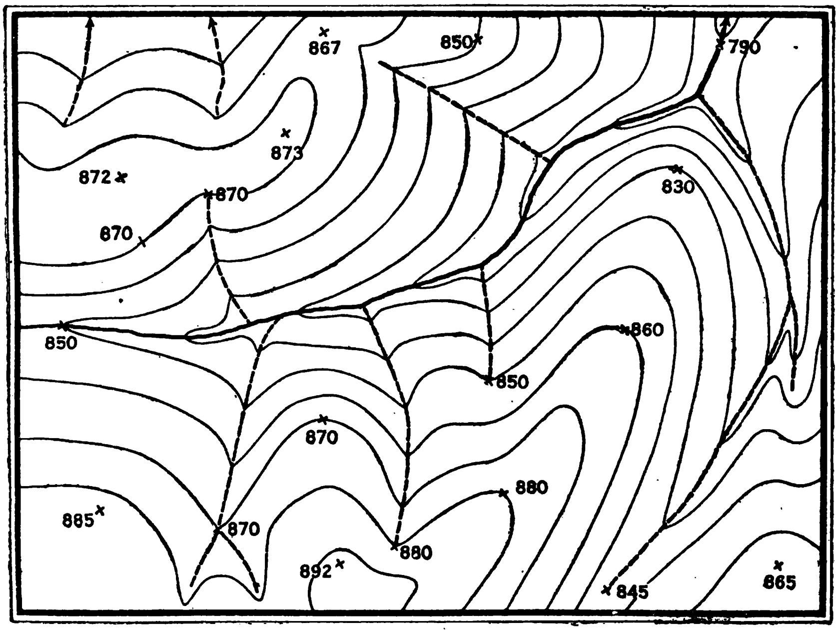


FIGURE IV

A stream may have at its source a very flat collecting basin, a lake, or pond.

(10) The *larger* the *stream,* the *flatter* the *slope* in the usual case. Hence, contours are usually closer together on tributaries than on main streams.

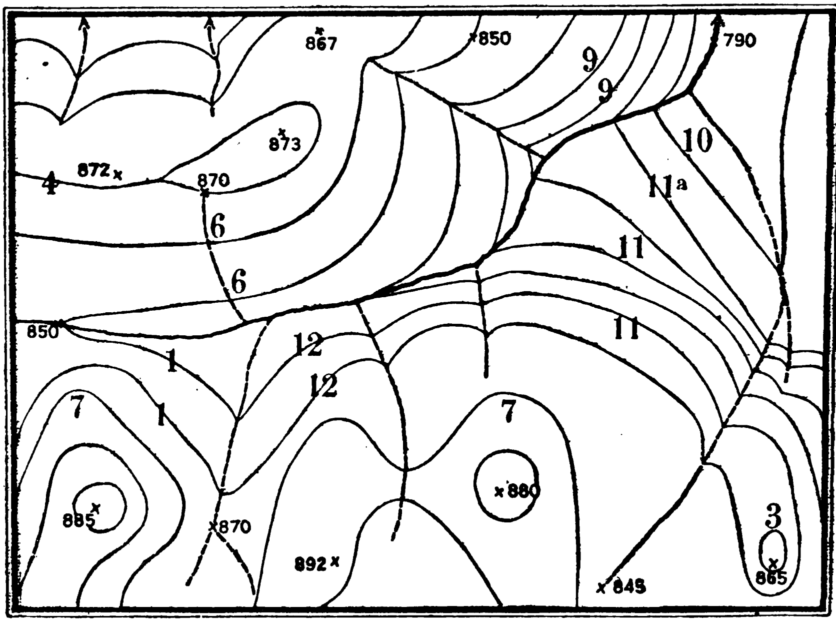
(11) Bad shaping of contours is usually due to illogical interpolation between critical points. Remember that the *drainage lines* and *ridge lines* are *master lines* of your contours. Interpolate along *drainage lines first,* beginning on the main *lines* and going to the tributaries. Then search out your *ridge lines* and *interpolate* along the *lines of the ridges.*

(12) If one has difficulty in tracing out a particular contour, he may be helped by imagining himself to be walking along the contour. If he starts out with *low land* on his *right hand,* he will always have *low land* on his *right hand* as long as he walks that contour in that direction and vice versa.

PRACTICE WORK.

21. Figures VI, VII and VIII are sketches giving the drainag net and critical points of three areas in the vicinity of Fort Leaven worth, Kans. The instructor will require each member of the clas to trace these skeletons one at a time and to draw in the contour following the systematic method heretofore described.

There can be no better training for one who is studying topograph sketching, or map reading. Not until one can complete a skeleto such as these, with great readiness, is he able to take up the readin of or sketching of topography. It is not considered desirable fo anyone to take up the study of Lesson III until he has mastere



MISTAKES IN DRAWING CONTOURS
FIGURE V

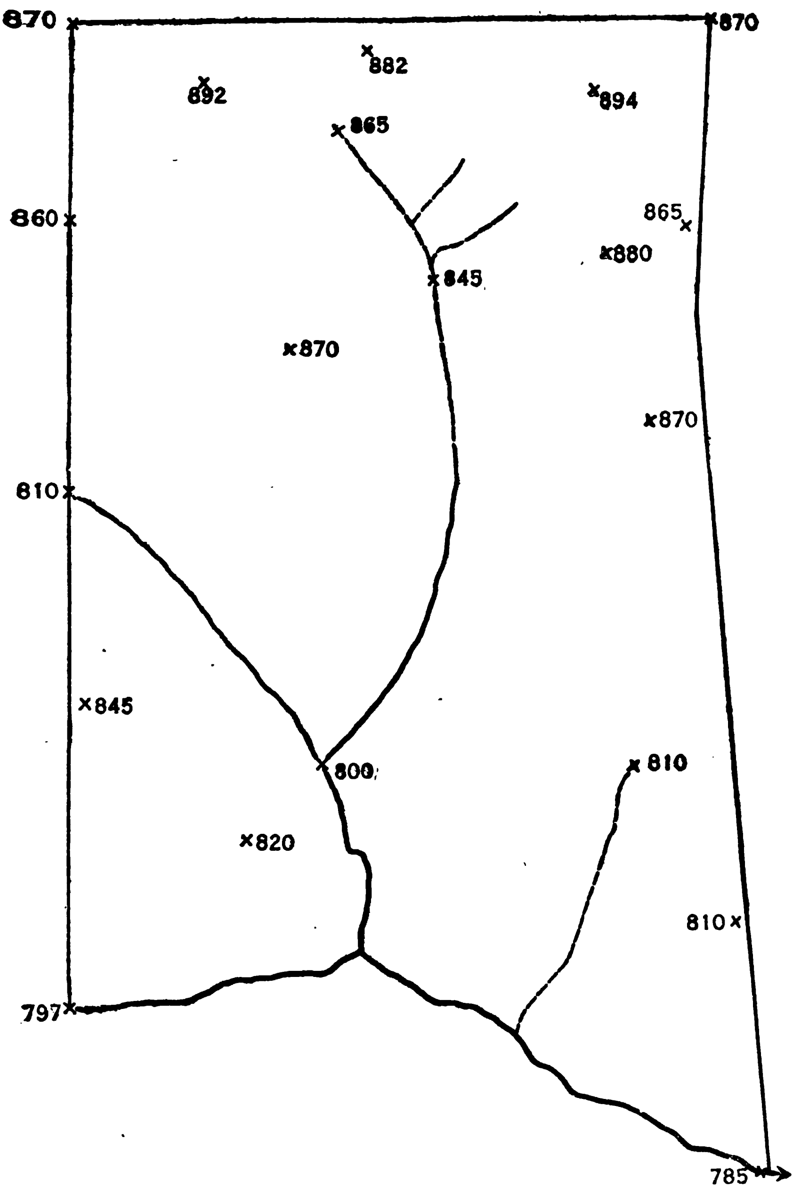
"logical contouring." It may be necessary, therefore, to devote an additional lesson period to practice in drawing contours.

LESSON III.

SKETCHING.

22. The instruments issued by the Engineer Department for sketching are based on the plane-table method. They are simple. Skill in their use can not, of course, be acquired without practice, but one can *begin* to use them with very little preliminary instruction.

It is the beginner's lack of a general plan of operations, his lack of appreciation of the essentials, his waste of effort and patience on the nonessentials, that make sketching the *bête noire* of a soldier's training, rather than the admitted inaccuracy of hand instruments and his lack of skill in their use.

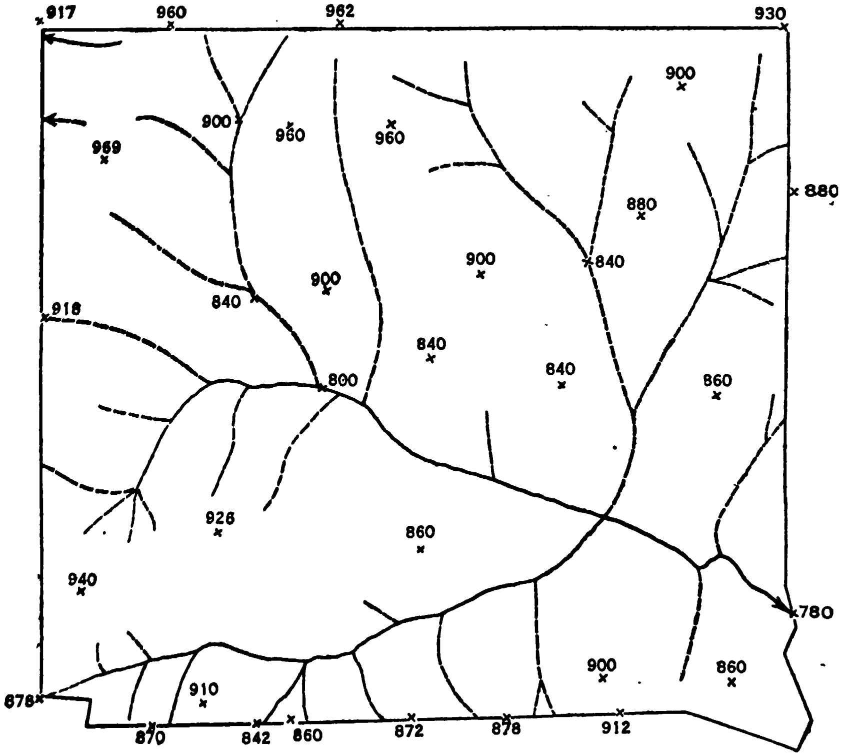


DRAW 10-FT. CONTOURS ON THE ABOVE SKETCH

LOWEST CONTOUR 790

FIGURE VI

The following plan of *operations*, if adhered to, will produce good work. It would be well if the instructor could select an area and lead his class around and through it as described, explaining the text to them as they proceed from point to point. The first *step* to be taken is to *prepare a skeleton* of the area entirely similar to those shown in Figures I, V, VI, VII, and VIII. The *next step* is to draw the *contours* and *finish* the *sketch* in all of its details.



DRAW 20-FT CONTOURS ON THE ABOVE SKETCH

LOWEST CONTOUR 780

FIGURE VII

PREPARATION OF THE SKELETON SKETCH.

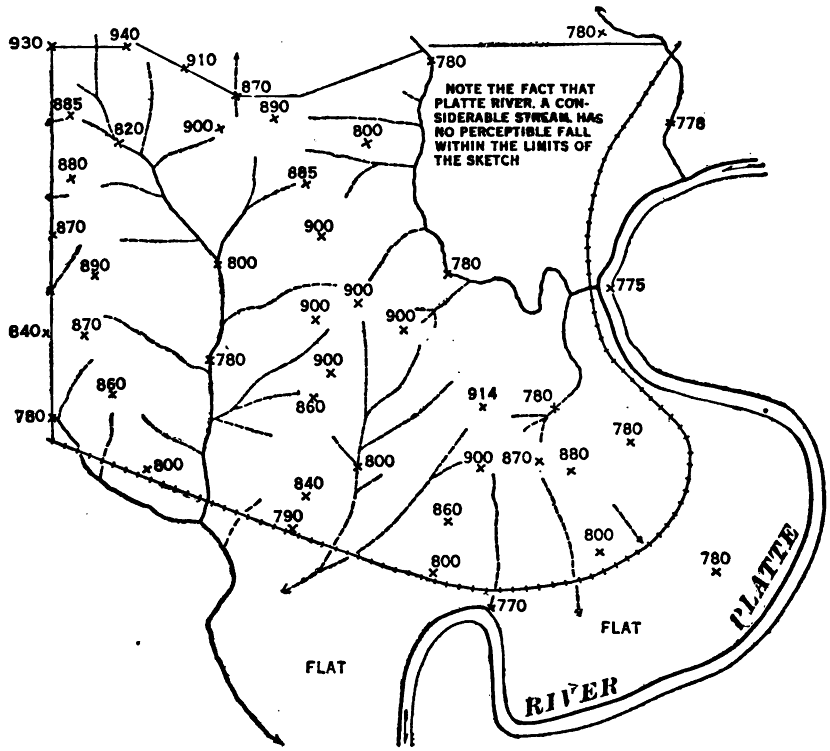
23. We have learned from our practice in drawing contours *what lines* and *what points* are essential on the skeleton. The problem now is how best can these be located. It would not do, of course, to attempt to *traverse* out every stream system and every watershed to its minute details. This would take forever, and would be contrary to the first principle of all mapping, which is to treat the area as a unit; "**proceed from the whole to the part.**" There are several good methods of procedure, but most of those usually explained are for the use of the man who has already become a good sketcher. The

following method is for the *beginner.* It may be modified as he becomes more expert.

24. **First, traverse around the area, returning to the point of beginning.** In traversing, locate—

(a) *Every drainage line* crossed or which runs along generally parallel to the line of the traverse and note the direction in which the stream flows (or the dry creek drains). This is absolutely necessary.

(b) *Every house* or other easily *identified feature* near *the road,* which will enable the sketcher later to identify himself on the sketch.



DRAW 20-FT. CONTOURS ON THE ABOVE SKETCH

LOWEST CONTOUR 780

FIGURE VIII

(c) *The high points* between drainage lines, i. e., the point on the ridge line where the traverse crosses the ridge line and a point or so on each side of the traverse on the ridge to locate the ridge line as well as may be possible.

(d) *Determine the elevations of all of these points. Do not draw any contours yet.* Even the experienced sketcher should be slow to put in contours until he has reconnoitered enough of the area to *pin down* the *drainage system. He must wait until he has seen enough of*

the country to know how the big features lie. The little ones will take care of themselves later as the work progresses.

25. **Second, adjust the traverse.** When in the circuit traverse the sketcher has reached a point from which the point of beginning can be seen, it will be found that when the instrument is oriented the ray to the point of beginning on the ground will not pass directly through the point of beginning as plotted on the paper. This is to be expected. If the error in direction is not out more than a degree, let it go at that for the time being and pace the distance to the point of beginning. Lay this distance off on the ray as actually drawn.

There will be an error of closure. If there has been no opportunity to check *orientation* or *distance* in the circuit, it will be good work if the closing error is not greater than 5 per cent of the entire length of the traverse. Providing the error in direction can not be accounted for by *local attraction at any point,* nor the error in distance by *incorrect* plotting of any course, *distribute the error* by moving points of traverse.

26. It will not be necessary to go into refinements in this adjustment. *Move the last point* up to the point of beginning; the next to last point but one in the same direction but a lesser distance, etc. **Be sure** *in doing this that no two courses have a* **plotted angle between** *them which differs noticeably from the* **actual angle** *on the ground; otherwise your sketch will mislead some one.* The sketch must have no *misleading errors.* It will have errors, but they must not be those which will mislead.

27. **Third, traverse through the area.** The sketcher has *now been around the area.* He has located all the *water* that *flows into* the area and all that *flows out.* He has *tied down* his *drainage system* pretty well. He will have noted the relative size of the streams or drainage lines and their elevations and this will give him a good idea as to which ones are the *main drainage lines of the areas.* He must now find out *how* these *drainage lines connect* up inside his area. In open country he may be able to locate the stream junctions in the interior by intersections from the traverse. Ordinarily, however, these stream junctions are hidden by tree growth or folds of the ground. It never does in sketching (at least for a beginner and seldom for an experienced man) to *guess* how the drainage runs through the area. *There is no royal road to stream junctions.*

If the sketch is to be a truthful report of a topographic reconnaissance, the sketcher must make the reconnaissance. Let him *traverse into* the *interior* and *find out* what is there.

Here is where the bad misleading mistakes of the sketch made by an untrained man are apt to be made. Contrary to all advice he has probably attempted to contour as he went along. He has drawn in a number of ground forms as they appeared to him as he went along on his circuit traverse. These forms are usually exaggerated in size. *Every hill looks big when one is on top of it.* His forms therefore project into his area much farther than they should. When he gets inside, he must either rub them all out or change his drainage lines, which he finds by reconnaissance to be *thus* and *so* to *so* and *thus* to correspond to his badly built ground. The result is worse than useless; it will deceive the user of the sketch.

28. The *manner* in which the *drainage connects* up *inside the area can only be found by going into the area* and contouring must wait on

this information. In entering the area follow a road or trail (the location of such features are needed anyway) and the sketcher traverses as he goes. *One traverse* across the area may not be enough; usually will not be enough, though if the line followed is well considered, short offset traverses from the cross traverse will serve to locate the *main drainage lines* of a small area (less than a square mile).

In these *cross traverses* it is sometimes best to follow the *main valley* and sometimes best to follow *the ridges.* The knowledge of the area as gained by the circuit traverse will help determine which is the easiest route to traverse to *get the stream junctions* and *ridge line points needed.* But any line at all will do if one is careful to run as accurate a traverse as he can and *remembers what he is looking for.* He must never run a haphazard traverse and be satisfied with the information found along it.

He must get off the line by offsets if necessary to get vital information such as stream junctions and stream heads or spur directions.

The beginner should never leave his instrument on the line of traverse and wander off to see what he can see. If he does he will plot it wrong when he gets back. *He must pick up the instrument* and take it with him, *traversing as he goes.* He may find a gold mine in the way of an outlook which will open up the entire countryside to his view and enable him to *check back* and insure his *orientation* and *distance.*

29. **Fourth, adjust.** Now the student has been *around his area* and has *been inside* and searched out and located his *stream junctions* and *ridge lines.* He has a skeleton much like the ones he has contoured by interpolation. But unless he has been adjusting his elevations all the time he will have some very peculiar level data. He will find that a stream which is 200 feet high on entering his area and 150 feet high as it leaves, has a most unaccountable elevation of 148 feet in the interior of his area. He will also have found on his first circuit that the elevations did not close and he will have adjusted this by dropping back. In other words, as he arrives on his circuit at the point where he began, he will find that he is 20 feet above the proper elevation. This is not bad if his circuit has been long. It is to be expected with a clinometer. Of course, he accepts the elevation of the point of beginning as correct; changes the last station by 15 feet, the next to the last but one by 10 feet, the next to the last but two by 5 feet, and leaves the rest as they are, or he may carry the distribution of the error down a little finer by taking in more stations and making the change more gradually, but any reasonable adjustment is all right, providing, of course, he maintains *elevations within sight of each other relatively correct.*

He must not in his adjustment make a stream *deep* where it is *shallow* or vice versa *nor must he change a hollow to a rise.* Similarly he must adjust his cross lines. When he crosses a stream line, two points of which he has already located in his circuit traverse, he must know that he should get an elevation of his crossing point corresponding to the relative distances to the two points already determined (unless there is in between some feature such as a waterfall or rapids or a dam. These things will discover themselves if the sketcher is on the lookout for them).

30. The reconnaissance sketcher has few, if any, points located for him with precision. He must, therefore, use all of his knowledge of ground, of water flow, of railroad grades, etc., to keep his work logical and consistent. He must know that large rivers do not fall more than a few inches to the mile (unless full of rapids); that within the area likely to be covered by a reconnaissance the fall of a large river will be so little as to be not readily ascertainable with reconnaissance instruments. He must know that any water-carrying drainage line (that is one carrying water the year around) is apt to have a lesser fall than a dry gully in the same vicinity and his acquaintance with the region being sketched will (if he observes) give him some idea as to the approximate fall per mile. He must know that as a reasonably true proposition tributaries have a greater fall than main water lines. His work must be continually required to answer such tests. If it does not he must be ready to give the reason.

CONTOURING THE SKELETON.

31. It is assumed that the student has now a logically correct skeleton of the drainage system and ridge lines. He must, of course, be sure that he has in all the roads and trails of military importance. He must not show these connecting up and running through unless he knows they connect up or run through and where they connect up and run. He must always remember that he is making a reconnaissance and reporting the result of his reconnaissance, not filling up inches on his sheet with conjectures. He must leave a blank if he *does not know*. The student should now take this skeleton and contour it without going back over the country. (I hear the objection, "I thought you should not draw what you do not see or can not remember. I thought one should never sketch in contours except in the face of facts.") Well enough, for the man who can do this, but the reconnaissance sketcher must learn. He can not follow the methods of the experts at once. By requiring him to contour his sketch, the data of which is incomplete, as of course it will be with the beginner, he will find out what essential data he has left out. He will find that he forgot to get the elevation of a hill, or the direction line of a ridge or a stream junction; he will have forgotten whether a certain stream had banks 10 feet high or 50 feet high; he will find that he left out the direction of the flow of a certain stream, and for the life of him will not be able to remember which way it did flow. All of this will teach him to put down his data in sufficient fullness so that he will be able to contour correctly from the data on his sheet, at least as correctly as the time spent on his reconnaissance will justify.

32. After the beginner has done a number of area sketches in this way he will have arrived at the time when he can sketch in form lines as he goes along (lightly, because they will be later rubbed out) and later can, after he has covered enough of the area to find the trend of the big features, complete his work as he goes along. But all of this presupposes that he has become so expert in his traversing that he will need to do little adjusting and that his eye for ground has developed to the point where he can see the forms and knows how to represent them on paper. Even then he will be tripped up occasionally and be obliged to rub out much work if he begins to contour

too early. One can contour as he goes along if he is working with accurate instuments and is sure that no adjustment will be needed later, but when working with reconnaissance instruments he is very apt to get into trouble.

LESSON IV.

33. In Lesson III there was described as fully as possible the procedure to be followed by a beginner in executing an area sketch. This procedure was given before the instruments and the technique of their use were described for two reasons:

First. It was felt that many of the officers and men under instruction will have had previous experience in the use of sketching instruments and that, therefore, they should proceed in the third lesson period as outlined.

Second. As the main purpose of this manual is not to teach the use of any particular set of instruments, but rather to teach the principles of topography, it was believed that the dry detail about instruments, scales, etc., might well wait until the class had absorbed a proper idea of what a contoured sketch is and what the procedure in learning to make one should be.

FIELD SKETCHING INSTRUMENTS AND THEIR USE.

34. The Engineer Department has designed a standard field sketching outfit, based solely upon the *plane table method* of sketching. The entire outfit packs conveniently into a carrying case. The supplies which are expendable consist of pencils, erasers, sketching paper, and celluloid. The celluloid is for use in the rain. The permanent equipment consists of a small plane table with folding wooden tripod, a service clinometer, a pace tally, an alidade, pencil pocket, which pins onto the service coat, and a holder for carrying timing pad in mounted work.

THE ALIDADE.

35. The alidade supplied is used for three purposes: (1) To determine differences in elevation from the scales of slopes provided; (2) for use as a sighting vane; (3) for use as a measuring scale, there being provided in addition to the scale of inches, blank spaces for pasting on individual scales of double paces, etc.

SCALE OF STRIDES.

36. Before proceeding to sketch it is of course essential for each sketcher to secure or construct a working scale. If the work is to be done dismounted the scale should be in strides (double paces). To prepare a scale of your strides for a sketch which is to be made 6 inches equal 1 mile, proceed as follows:

Measure off a rating course of 1 mile, $\frac{3}{4}$ mile, or $\frac{1}{2}$ mile. The longer the course the better. It should be over average ground (up and down hill) similar to that which is to be sketched and should be carefully measured. Walk this course both ways several times, counting strides with the pace tally. Take the average for this distance and from it find the number of strides to 1 mile. Say the

number of strides for 1 mile is 1,048. The scale of the sketch is to be 6 inches equals 1 mile, therefore 6 inches on the map will represent 1,048 of your strides. If you divide 6 by 1,048 you will have the scale length for one stride. It is too small a division to be useful, so multiply by 100 and find length for 100 strides is 0.57 inch. Pages 69 and 70 show working scales for the ordinary range of topographic sketches.

With a scale of equal parts (one reading inches and tenths will do) lay off as many divisions as you may desire in your scale. Each of the divisions will represent 100 of your strides. Divide the left division into 10 equal parts with your eye. Each of these will then be 10 of your strides. No smaller distance can be well measured off on your sketch.

SLOPE SCALES.

37. The alidade, as has been mentioned, in addition to the blank spaces for the individual stride scales, is provided with slope scales. (These are sometimes called *map distance* scales, which is confusing.) These scales are used in sketching, in connection with the clinometer, to find the differences in elevation between two points, the vertical angle from one to the other having been read by the clinometer.

The slope scales give results which are sufficiently accurate for sketching purposes. They are not reliable for angles greater than 15°

Each interval on the slope scale represents the distance on the sketch (scale 6 inches to the mile) corresponding to a rise of 10 feet for the slope given. Working on a 6-inch scale, therefore, the method of determining the difference in elevation between two points is as follows: From the point of known elevation, read the slope to the point the elevation of which is to be determined. Say, it is 3°. Pace to the second point to find the distance and plot the distance. Look on your alidade for the slope scale of 3°. (The figure representing the slope is engraved at the midpoint of the interval for that slope.) Apply the slope interval for 3° to the plotted distance and find out how many times this interval is contained in the plotted distance. Multiply this number and fraction (there will usually be a fraction, which must, of course, be estimated) by 10 feet and the result is the difference in elevation. Similarly, for other clinometer readings, take the slope interval corresponding to the clinometer reading and find out how many times it is contained in the plotted distance, and then multiply by 10 feet.

38. This same slope scale may be used for any scale, providing that instead of multiplying the number of intervals and fractions thereof by 10, we multiply by the rise for that scale. The rise for any given scale is found by dividing 60 by the number of inches which represents the mile. For instance, the rise for a 3 inch to the mile scale is 60/3, or 20 feet; for 4 inches to the mile scale it is 60/4, or 15 feet; for ½ inch to the mile scale, 60/½, or 120 feet. The method is the same, the slope scale is the same, no matter what the scale. The only thing that changes is the rise per interval, and this rise per interval is found as indicated above.

39. It will occur to the instructor or to some of the sketchers that to hunt around the alidade for the slope scale corresponding to a particular clinometer reading is rather a nuisance, and so it is.

This difficulty may be obviated by constructing "a scale of differences" and pasting it on the blank space of the alidade not used for the stride scale. It is constructed, marked, and used as follows:

On a suitable piece of paper lay off a line; on this line lay off several intervals equal to the 1-degree slope scale interval. These are about 0.65 inch each. Mark the division points, 10, 20, 30, 40, etc. Divide each interval into 10 parts (the 10-degree slope interval will do this about correctly), cut and paste on alidade. If working on a scale of 6 inches to the mile, this difference scale is used as follows: Read the vertical angle with the clinometer. Pace the distance. Plot the distance. Say the clinometer read 1° exactly; apply the "scale of differences" to the plotted distance between the points and read off directly the difference in elevation. If the clinometer read 2½°, apply the scale as before, but multiply the difference in elevation as secured directly (which is that for a 1° angle) by 2½° to secure the difference in elevation for a 2½° vertical angle. This same "scale of differences" may be used for any other scale in sketching by simply multiplying the result as above secured by the proper scale factor. The proper scale factor is found by dividing six by the number of inches to the mile in the scale being used. For a 3-inch scale, multiply by 6/3, or 2; if working on a 2-inch scale, multiply by 6/2, or 3. Such a scheme saves time and worry and is comparable in convenience to using a percentage clinometer and a per cent scale.

It will be seen that the method simply makes universal use of the 1-degree interval, instead of a different interval for each vertical angle. This is not too inaccurate for sketching use.

Of course, if using another scale than 6 inches to the mile for any considerable amount of work, the "scale of differences" can be remarked to correspond. That is, change the readings on scale by multiplying by 2 for a 3-inch scale, by 3 for a 2-inch scale, etc.

SERVICE CLINOMETER.

40. To use the clinometer, the observer sights the object through the peephole and, at the same time, sees the scale of degrees inside the rim. The red figures are minus (depression angles), the black ones plus (elevation angles) readings. The pendulum is allowed to swing freely by sliding back the bar and pressing the stop. The reading, is taken when the desired point is sighted and the pendulum has come to a rest. The pendulum should not be stopped to take the reading as this may displace the scale. It is best to rest the instrument or the hands holding it against a tree, fence, post, or other object while taking a reading. Theoretically the reading should be made to a point about the same height above the ground as the observer's eye. Actually this is very difficult to do and it is best to always read to the ground line and then, after ascertaining the difference in elevation, make allowance for the height of the eye above the ground. The clinometer should be tested occasionally as follows: Hold the instrument, stop uppermost, against a post or tree (point marked by a piece of paper pinned to it) and read to another point marked on another tree, 50 or 60 feet distant, and note the reading. Reverse this operation and read from the second mark back to the first one and note reading. For example, the first

reading is minus 2°, the reading back is plus 1°. Add the two readings and divide by 2 and you have the correct reading (1½°) of the slope between the two points. The clinometer then reads ½° too much on minus and ½° too little on plus readings. Reading must be corrected accordingly.

41. **The sketching board** is a simple board with a magnetic needle set in a trough in one edge. The needle is 3 inches long and quite sensitive. No plumb bob is provided, but a hole is accurately bored so that a plumb bob can be improvised and use made of the slope scale on the board in case the clinometer is lost. The plate on the back of the board is let in flush, so that the board can be turned freely on the tripod for orientation, and then firmly clamped by a slight turn of the tripod screw. As the tripod is not used in mounted work holes have been bored at the corners of the board for the insertion of a carrying cord if desired.

TO ORIENT THE BOARD BY COMPASS.

42. Set up the tripod with board loosely screwed onto it and level the board by eye by moving the tripod legs. Free the needle by turning the cam and then turn the board slowly around until the needle swings from side to side in the trough. Let the needle settle, turning the board as necessary so that the needle when settled lies directly along the north and south line (the median line of the trough). Without changing the position of the board reach under and tighten up the screw of the tripod. Care must always be taken not to turn the tripod screw too tight, as the threads are likely to be started by rough treatment. The board is now oriented. Draw a line parallel to the needle on the paper and mark the north end with a half arrow. Above this write M. M. or N. (magnetic meridian or north).

TO ORIENT THE BOARD BY BACK SIGHT.

43. Having plotted (located and drawn in) a station and arrived at a point farther on to which you have sighted and drawn a line, set up the tripod and board, measure off the number of strides taken between the two points on your line and stick a pin in the point found. This is your present position. Place your ruler against the pin and along the line between the two points and turn the board until the station you have just come from is sighted. Tighten up the tripod screw without moving the board and you are oriented. Verify this by reading the needle.

TO LOCATE A POINT BY TRAVERSING.

44. By this is meant measuring the distance between two points, by counting strides, or time or travel required to pass from one to the other. The term traverse is applied to the route followed by the sketcher in making the sketch. Being at point A (whose position is plotted on your sketch), with board oriented, stick a pin in point A on the sketch. Lay alidade alongside the pin and pivot it around until point B (the point to which you are to traverse) is sighted. Verify the position of the needle and then draw a ray toward B. Move to B, counting strides with pace tally, and upon arrival set up

the tripod, orienting roughly by a back sight on A. (Do not waste any time on this.) Lay a ruler along the ray, zero of scale of strides at A, and lay off on the ray the number of strides that your tally register shows was taken between the two points. The point marked is B.

TO LOCATE A POINT BY INTERSECTION.

45. Assume that from point A, with your board oriented, you took a careful sight and drew a ray toward a church a few hundred yards off to the side. After arriving at B and plotting its position, you carefully orient the board. Pivot the ruler around the pin in B until the church is sighted, then, verifying the position of the needle, draw a ray toward the church. The intersection of this ray and of the one you took from A is the sketch position of the church

TO LOCATE BY RESECTION.

46. This is determining the sketcher's position (the reverse of intersection) by orienting the board and drawing rays toward himself from two or more points whose positions are already determined and plotted. Having previously plotted the position of points A and B, the sketcher comes into his area later, at some point from which those two points can be seen. His present position is not yet determined. To determine it, set up the board and carefully orient by compass, then stick pins in the sketch positions of A and B. Pivot the alidade on the sketch position of A, sight A, and draw a ray. Do the same with B. The intersection of the two lines is your position

TO RESECT FROM THREE KNOWN POINTS.

47. In the previous paragraph a method of resection from two known points was given. For this a needle orientation is necessary. At times local attraction may deflect the needle and the three-point resection may be useful. The simplest method is by use of a transparent or translucent paper (celluloid will do splendidly). The process is as follows: Pin the paper on the board by thumb tacks. Stick a pin in it anywhere at a convenient point to represent your station (pay no attention to what may be on your sketch). Pivoting on the pin, draw, with great care, rays to each of three points which are in view and which also appear on your sketch. Lift the paper off the board. You have on it three rays, which make with each other the proper angles, considered from your position. All you need now to do is to put the tracing paper over your sketch, and by looking through to your sketch shift the paper until each ray passes through the sketch representative of the object to which this ray was drawn. As soon as each of the rays passes through its proper point (which will take a deal of fussing around the first time) you have it. Prick the point of intersection through to your sketch and the pinhole will give you your plotted position. This has been all done without orienting the board. If we wish to work from this station, the next step is to orient. This is done by back sighting to one of the three objects. Remember, this is but a cut-and-try method. It may take considerable time. It may help much at times.

LESSON V.

48. The class by this time has progressed to the point where a great majority know what to look for in making an area sketch and know how to use all of their sketching instruments. It is now believed essential, at the risk of some repetition, for the instructor to give a special talk on the technique of the *sketcher's traverse,* the *detail* which should be shown on the sketch, and a few hints which may make for *clearness.*

TECHNIQUE OF A SKETCHER'S TRAVERSE.

49. Set up the board at the point of beginning, station 1. Free the needle. Swing the board until the needle comes to a rest along the median line of the declinator trough. The board is now oriented. Clamp the board by tightening the screw holding the board to the tripod. The billiard-cloth covering of the tripod head will hold the board in position. Do not use an excess of force in clamping. Plot a point (use a pin to pivot on) on the paper to represent the point of beginning. Place the alidade on the board. Pivot it on the pin representing station 1. Sight along the line to be traversed to station 2. It is advisable where possible to sight on some well-defined object, as a tree, fence, corner, etc. This is necessary when the course from 1 to 2 is short. If the course is a long one, and follows a well-marked road or trail, a sight along the mid line or edge of the road will serve. Draw the ray. Pencil should have a chisel point and should be held vertically so that the pencil line will follow edge of alidade as closely as may be. *Always see that needle is along mid line of declinator trough when ray is drawn. Put in local data* around station 1. Not necessary to draw rays to all points to be plotted. This fills up the sketch with lines. Point the alidade at object, and having ascertained the distance to it by pacing or by estimating plot the object in its proper place at once. *Do not plot any points ahead of station 1 along the line of traverse. Draw rays to well-defined, easily identified distant objects, noting on the ray the object to which drawn.*

A ray to these same objects from another point on the traverse will serve to locate the objects, and once located they will serve as excellent checks upon orientation or may serve to locate the instrument by resection when for some reason or other the location is otherwise unknown. The tendency with the beginner is to draw rays to everything in sight. This is to be discouraged, as he loses much time in getting the rays—usually gets them confused so that all are useless—and seldom is he able to identify more than a very few of the objects so sighted. Read the clinometer, reading from station 1 to the objects sighted, noting the slope on the ray. Read the clinometer to station 2 last of all. Most texts tell you to sight at a point at station 2, which is the same distance above the ground as is your eye in taking the sight. It is much better to pick out a definite point to sight on. In other words, sight at the top of the chimney of a distant house, the lower window sill of a nearby house, the base of a tree, or, failing anything else, the ground line. Now, the slope you have read is the slope from your eye to the point sighted. You know the elevation of your eye. After getting the

difference in elevation between eye and object sighted this difference is added to or subtracted from the elevation of your eye and the result is the elevation of the object sighted. From this the elevation to the ground level at station 2 is found.

50. **Set your pace tally to zero**, pick up your equipment, and start off in the direction of station 2, walking with your customary step. Do not try to make your steps uniform. Walk naturally. Your scale should have been made to read double strides while you were walking naturally over country similar to that which you are to sketch. Press the pace tally every time your right foot strikes the ground, pace tally in your left hand. Whenever you reach an object which should appear on your sketch, as a house, bridge, stream line, ridge line, etc., do not take the time to set up and orient unless necessary. This will only be necessary when you wish to get a direction with care, as in the case of the direction of an important stream or ridge line. If the object is a house close to the roadside, it should be plotted in at once without set-up or orientation.

51. **Arrived at station 2**, set up. Scale off number of strides shown by your pace tally between stations 1 and 2 on ray already drawn. Mark station 2 by small circle with dot in center; elevation as soon as found beside the circle. **To find elevation of 2.** Sight back to 1 with clinometer. If reading is not the same as it was from 1 to 2, take the mean (providing of course line of sight is from ground to ground, or allowance has been made for elevation of eye in each case). Now hunt for the proper slope scale for the slope read. Apply the interval to the plotted distance from 1 to 2, to find the number of intervals and fractions thereof contained in the distance. Multiply this number by the rise per interval, 10 feet for 6-inch scale, 20 feet for 3-inch scale, etc. As soon as station 2 is plotted and its elevation found, orient board by compass and check by back sight. Then proceed as before. (The elevations of all points on the traverse between 1 and 2 should of course be noted as well as the elevation of the side shots taken. These are found in the same way as that of station 2.)

In plotting distances avoid as much as possible scaling off long lines by short plotted distances. In other words, keep track of the number of strides from the beginning point of a long tangent or slightly curving road and locate all stations on the tangent by scaling from the first point. This means that the pace tally is only set back to zero at a marked change of direction. Watch your pace tally to see that it does not fail to register.

DETAIL AND CLEARNESS.

52. The principal aim of the course so far has been to instruct the student officer in ground forms and the method of representing them on a map by contours. Every effort has been made to impress upon him and the instructor alike the necessity for viewing an *area* of *ground* as a *unit;* the necessity for *searching out* the *system* of the *topography* before minor matters are taken up.

In the sketching which has been done by the class, therefore, every effort has been made to *compel* each individual to *get around* and *through his area* so that he might pin down the *drainage system* if nothing else. Upon this rather inadequate framework he has been

compelled to draw in contours at very small vertical intervals. In putting in the troublesome contour lines he has very likely rubbed out roads, houses, most of his fences, and all of his trees. The resulting sketch is apt to be characterized more by its intense *blackness* than by its clearness and military value.

A military sketch must show the *essential military* features of the area and as many of the less important military and other features as the scale of the sketch and the skill of the sketcher make possible. Just what amount of detail should be shown on a sketch in any particular case is a matter of judgment. As one's skill in sketching and training in military tactics proceeds, his ability to determine these questions for himself will grow.

53. A sketch to be of value must be clear. The essential features must be brought out with bold strokes. Do not fill up the sketch with rude conventional signs for trees, grass, and corn. Do not fill it up with a mass of crowded contour lines. While the normal system requires 10-foot contours for a 6-inch scale, and 20-foot contours for a 3-inch scale, this system is not to be followed slavishly. If the country is so steep that 10-foot or 20-foot contours will crowd each other, use a larger interval.

Break contours at the railroads and *roads*, so that these important lines of communication will stand out. Break them at cliffs and write the words "cliff 60 feet high." Break them at streams and write "stream 50 feet wide, banks 40 feet high, unfordable except as indicated." Omit no important *landmarks*.

54. The user of the sketch must be able to locate himself on the sketch and for that reason landmarks must appear upon the sketch at frequent intervals (if they exist in fact). A tumbled-down shack in a sparsely-settled region, a group of cottonwood trees in a treeless waste, a waterhole in a semiarid country, a group of large buildings of a public or semipublic institution in the vicinity of a town will serve to help the user locate himself. A stone bridge may name a battlefield.

55. The conventional signs as adopted for use by all United States Government departments, as well as the special sketching signs given in the Field Service Regulations, follow paragraph 155. They must be learned.

56. At least three lesson periods must now be spent in sketching.

LESSON VIII.

MAP READING IN THE FIELD.

57. Map reading is essentially the reverse of map making. One who has been practiced in sketching by the plane-table method should have no difficulty in reading and using a large scale sketch of a small area. The essential features stand out almost at a glance. But a map of a large area of ground can not be read at a glance. It must be measured and studied in much the same way as the ground itself would have to be measured and studied before we become familiar with the country represented.

Scale.—As in making a sketch we need a "*working scale*," so in studying a map we need a "*reading scale*." It is not sufficient in *reading* a map for military purposes to glance at the scale given on

the map and then to glance at two points and guess at the distance between them. The *units* of distance most used in our service are the *yard* and the *mile*. It will be useful, therefore, to prepare our *reading scale* in yards or miles. Pages 67 and 68 give scales for the usual range of topographic maps.

58. **Examples.**—To prepare a *reading scale* of *yards* for a map on a scale of 1:80,000. Here 1 yard on the ground is represented by $\frac{1}{80{,}000}$ of a yard on the map. 1,000 yards on the ground by $\frac{1{,}000}{80{,}000}=\frac{1}{80}$ of a yard or $36/80=0.45$ of an inch. Lay off a line 4.5 inches long and divide it into 10 parts and you have your scale. If a smaller unit than 1,000 yards is desired, divide the left unit into 10 equal parts.

59. To prepare a reading scale of miles for a map on a scale of 1:100,000. Here 1 mile on the ground is represented by $\frac{1}{100{,}000}$ of a mile on the map. One mile is 5,280 feet, or 63,360 inches. Therefore the map representation of 1 mile on the ground is $\frac{63{,}360}{100{,}000}$ or 0.634 of an inch. Lay off a line 6.34 inches long. It will represent 10 miles. Divide it into 10 equal parts and each part will represent 1 mile.

ORIENTATION AND DIRECTION.

60. To one who does not know his own position on the map or who does not hold it properly oriented when in use, a map is a hindrance rather than a help. One who is responsible for guiding troops by a map must keep his position on it by constant reference to the map. Too frequently in maneuvers, and even in actual war, is the map hidden away in a dispatch case. Then, after the column has already gone astray, frantic efforts are made by all concerned to recall the various textbook methods for orienting a map and for locating one's position upon the map.

PRACTICE WORK IN ORIENTATION.

61. It is useless to describe methods in a training manual unless the class is actually practiced in them. An indoor reading of the method of making a sketch or of orienting a map will do a beginner little good.

The following methods of orientation and resection should be tried out in the field by the class under the eye of the instructor:

First method.—Take a map of the training camp with the magnetic meridian marked upon it. Set up a sketching board on its tripod. Put the map on the board. Shift it until the magnetic meridian on the map becomes parallel to the meridian line of the needle trough. Pin the map down with thumb tacks. Orient the sketching board and resect from two points in view as described in paragraph 46.

Second method.—Using a compass, but not the sketching board. Lay the map on the ground. Lay the sight line of the compass along the magnetic meridian of the map. Rotate map and compass together until needle points north. If sight line of compass has

been kept along magnetic meridian of map, the map is oriented. Your location may be found by resecting as described in paragraph 46.

Third method.—No compass or no magnetic meridian. True meridian given; position unknown. Point the hour hand of your watch (held face upward) at the sun, if in the Northern Hemisphere. The line drawn from pivot to the point midway between the outer end of the hour hand and XII on the dial will point toward the south. Shift your map to correspond. This will give a very rough orientation. You may now be able to identify two or three distant points on the ground and their representations on the map. If you can do so, resect by the method explained in paragraph 46 or paragraph 47. Your resection will not be accurate, but it will serve to aid you in locating yourself.

62. While the last method is a good instruction method, there is no excuse in campaign for an officer not to have the magnetic meridian on his map. If the magnetic meridian is not shown on the map, ascertain the magnetic declination and draw the meridian on. The magnetic declination, it must be remembered, is the angle which the magnetic needle makes with the true north at the point. If the declination is east, it means that the north end of the needle settles to the east of true north. If west, the north end of the needle settles west. If the *magnetic declination* can not be readily ascertained, you can determine it approximately in the following manner:

DETERMINATION OF MAGNETIC DECLINATION.

63. In the explanation of this method, the term magnetic azimuth as here used is the horizontal angle from the north point of *the needle* measured *clockwise around the circle* to the *object* sighted. To read azimuths correctly, a box or pocket compass should be graduated *counter-clockwise.* If yours is not so graduated, better add a rough graduation in the counter-clockwise direction.

Observe the magnetic azimuth of the sun, a planet, or a bright star at *rising* and *setting* on the same day or at *setting* on one day and *rising* the next. Add these two azimuths together. Take the difference between this sum and 360°. One-half of this difference is the declination of your compass—east, if the *sum* of the *azimuths* is *less* than 360°; west, if it is *greater.* In using this *method* the observations are best taken when the object is *just above* the *true horizon,* or at a *gradient of zero.* This can usually be done if a high point is chosen for observation. *If this can not be done,* be careful to take both observations with the object *at the same* gradient (as determined with clinometer). *This is most important with the sun.* Under the least favorable conditions an inequality of 1° in the gradients at the time of observation on the sun may introduce an error of ½° in the result.

In using a star, choose one which rises nearly east from the point of observation. If this be done the inequality of a degree in the gradients will be immaterial. Both observations need not be made at the same point, but should not be more than 10 miles apart in east and west or north and south directions. (See also par. 68.)

LESSON IX.

DAY GUIDING BY MAP AND COMPASS.

64. The following condensed quotation from a recent supplement to the British Manual of Map Reading and Sketching is taken from the International Military Digest, March, 1916:

After mastering the *conventional signs*, map reading is only a matter of observation and *common sense*, a fact which leads people to underestimate its difficulty and the need for *continual practice*.

When finding the way by road, it is important to consider the *time factor* in attempting to identify points of the road in advance, and *not place sole reliance* upon *cross-roads*, or *side roads*, etc., which may have changed since the map was published. The speed of march may be obtained from a speedometer or by a rough approximation of the marching rate of the column. * * *

Orienting the map by pointing at the sun [a common textbook method and at times a valuable one] may lead to an error in the direction of as much as 7° or 8°. Therefore, some *permanent point*, as a *church steeple* or a *straight* piece of road, should be sighted on.

When a *point* of the *country* has been identified on the map, time often will be saved by using it as a *reference point* in referring to others. Such points may be noted as so *many degrees* from it, or in line with it, or just to the right or left. A protractor will assist in laying off angles and measuring bearings on the map.

On a small-scale map many details are not shown, but much may be inferred from what is shown.

In *night movements practice* is *essential*, and although it is not difficult to march by compass bearing, much assistance can be obtained by the ability to recognize a *few stars* in *different parts* of the *heavens*.

65. After the remarks given above have been discussed, the instructor should practice the members of the class in following a course along roads and across country from the map. In order that this practice may not be done in a perfunctory manner, it is desirable to require the student officers to traverse a course outlined by the instructor, and note on the map all features which may be altered. If the map is so nearly correct that this method is impracticable, flags bearing special numbers may be erected along the course and the student officers required to locate the flags on their sheets. It will be particularly good practice if a group of flags representing a trench are so located that they can be found only by following a compass bearing across country for a distance of several miles.

Such practice will convince all of the great difficulties in guiding by compass alone. A judicious use of the map and of prominent physical features will simplify the matter in open country.

Before starting, the course should be carefully protracted onto the map and note taken of the *prominent physical features through which* or *near which* the line passes. Before starting, identify one of these points on the ground and march on it. If you must descend into a valley to lose sight of the feature, take some point in the valley which is in line to march on. Thus, continue from feature to feature. In close country this is not so simple. Here it may be necessary for you to actually plot on your map, or an enlargement of it, your course as you go. Make your courses as long as possible.

LESSON X.

NIGHT GUIDING BY MAP, STAR, AND COMPASS.

66. The following is taken from the Artillery Journal:

As an aeronaut in Ladysmith, I had plenty of opportunities of foreseeing the great power aeronautics would have in warfare in the future, and that most of the effective fighting would be done at night..

At that time the regulations described night operations as extremely hazardous, and warned the commander who undertook such operations that he did so at his own peril and was responsible for the results. Various expedients were suggested to enable the troops to keep their directions, such as that the route should be previously reconnoitered and marked by tins, pieces of paper, and other devices; but how the reconnoitering party was ever to carry out this operation nobody has yet been able to understand. We found the *colonials* never required this artificial help, and could move about on a *starlit night* as easily as in *daylight* and as fast as the nature of the ground permitted, * * * and ascertained from various colonials, Basutos, Indians, and Arabs that they could instinctively *read* the *heavens* as a *compass*, this knowledge having been transmitted from father to son for generations. * * *

Although the system was only perfected in June, 1915, *soldiers* of all ranks have begun to realize the simplicity and wonderful utility of being able to read the *universal compass*, the *heavens*, and we begin to hear how useful this knowledge has been found for guiding supporting troops up to the first-line trenches, etc.

The heavens can not go wrong, and on a starlit night you can rely absolutely upon them to take you to your destination, *once you grasp* the rudiments of the system; you only *require* to know three or four first magnitude stars, for their exact position is given for every hour of the night in Marching and Flying without a Compass (Tilney, lieutenant colonel, F. R. C. S.)

TO FIND THE STARS.

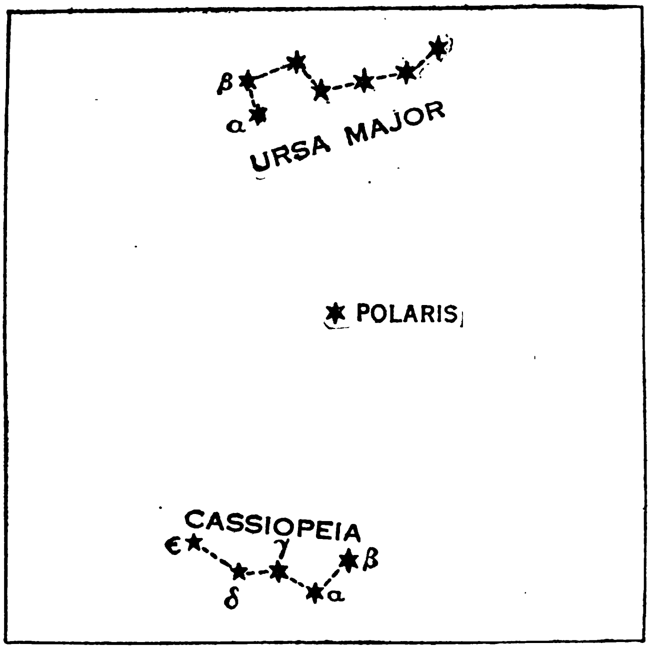
67. *The Dipper and Cassiopeia.*—The star plans given here, and parts of the descriptions, are taken from The Star Pocket Book by Capt. Weatherhead, British Royal Navy.

Ursa Major (called the Big Dipper), shown in the upper portion of Figure IX, is the most important of the constellations. It is at once the easiest to distinguish, the easiest to find the North Star by, and the best *starting point* from which to learn the other stars. The "Pointers" α and β point to **Polaris** (the North Star) at all times as the Dipper circles the Pole. On the opposite side of Polaris and at about the same distance from it is the constellation of Cassiopeia. Its form is that of the letter *W*.

The great importance which attaches to Polaris (the North Star) is that it is never more than 2° away from the point where the axis of the earth if extended would pierce the heavens. It therefore appears to the eye to be always in the same place, and it is, except for a maximum variation of about 2°.

68. As the *latitude* of any place is equal to the *altitude* of the Pole, when the Dipper and Cassiopeia are on either side of the North Star (east and west), the elevation of the North Star will give a reasonably correct figure for the latitude of the place of observation. When the Dipper and Cassiopeia are above or below the North Star, a compass reading to the North Star will give the *declination* of your *compass* to within the *least reading* of your *compass*.

69. *Arcturus, Spica, Denebola.*—The lower half of Figure IX shows the *Big Dipper* again. By continuing the sweep of the *Dipper handle*, you will find the bright star Arcturus about 30° from the end of the handle. Continue on with the curve and you will find *Spica* about 30° farther on.


URSA MAJOR
β
α
POLARIS
CASSIOPEIA
ε
γ
β
δ
α

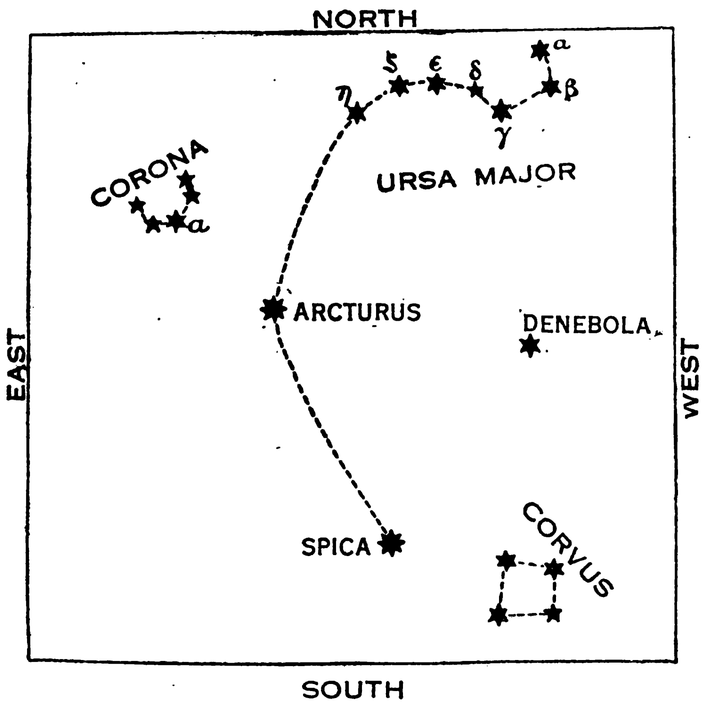

NORTH
EAST
WEST
SOUTH
URSA MAJOR
α
β
γ
δ
ε
ζ
η
CORONA
α
ARCTURUS
DENEBOLA
SPICA
CORVUS

FIGURE IX.

Arcturus has a good marker near it in the semicircle of stars called the *Northern Crown,* and *Spica* has the kite-shaped quadrilateral Corvus.

Denebola forms with *Arcturus* and *Spica* an equilateral triangle.

70. As an additional aid in finding these first three important single stars, it is pointed out that if *a star* is to be found at a *certain place* in the heavens at a *certain hour* on a *certain night,* it will be found in that *same place* the *next night about four minutes earlier* and on the next *four minutes earlier still,* or at the end of a month in that same place about *two hours* earlier, and so on for each successive month. Now, the "Pointers" (α and β of the Big Dipper) are to be found high on the meridian at midnight on March 7 any year. At this time *Denebola* will also be near the meridian (a little to the east) and in the south. Follow the sweep of the Dipper handle and high up in the heavens almost to our east will be Arcturus, with Spica still on the sweep of the handle in the southeast, completing the equilateral triangle.

71. Hold the chart directly above your head, north to north, west to west, etc., and you should have no trouble in finding these stars. Remember the stars are to be found in this portion of the heavens and as stated at 10 o'clock on April 7, at 8 o'clock on May 7, etc. After May 7, therfore, we may expect to find them getting lower and lower in the western heavens as the summer advances until September, when the Dipper is to be found low down under the Pole; Denebola and Spica will have set before night comes on.

72. In looking for the stars during the *summer months,* therefore, face the west and hold the chart in front of you, north to the north and *east side* up. See table in paragraph 77 for the date on which *Arcturus, Spica,* and *Denebola* transit at *midnight.*

73. *Vega, Altair, and Deneb.*—The upper half of Figure X shows the Dipper once more and the important stars *Vega, Altair,* and α Cygnus (also called *Deneb*).

Vega is to be found on the meridian at midnight July 1, Altair July 19, and Deneb August 2. They all, therefore, transit within about two hours of each other. There should be no difficulty in finding the T of Cygnus and from it Altair, and then *Vega,* which is on a line with *Altair* and its two accompanying stars.

In the early summer months, in the hours before midnight, we will find these stars in the eastern heavens. In the fall we will look for them in the west.

74. *Perseus, Andromeda, the Great Square, and Pleiades.*—In *line with the "Pointers"* on the other side of the North Star, and beyond Cassiopeia about as far as *it* is beyond the North Star, we find α and β of Pegasus. These five stars are all on a line (great circle), and all cross the meridian together, but, of course, when the *Big Dipper* is high in the heavens, *Pegasus* being on the other side of the Pole is out of sight in ordinary latitudes. When Pegasus is high in the heavens, the Dipper is below the North Star. It is to be expected, then, that α and β Pegasi will be on the meridian at midnight on September 7, just *six months* after the *"Pointers"* made their *upper transit* at *midnight* and on the *same night* as they make *their lower transit* at midnight.

The stars on these charts, therefore, are to be looked for in the early hours of the summer evening in the east; high in the heavens in the early fall; and in the west along Christmas time.

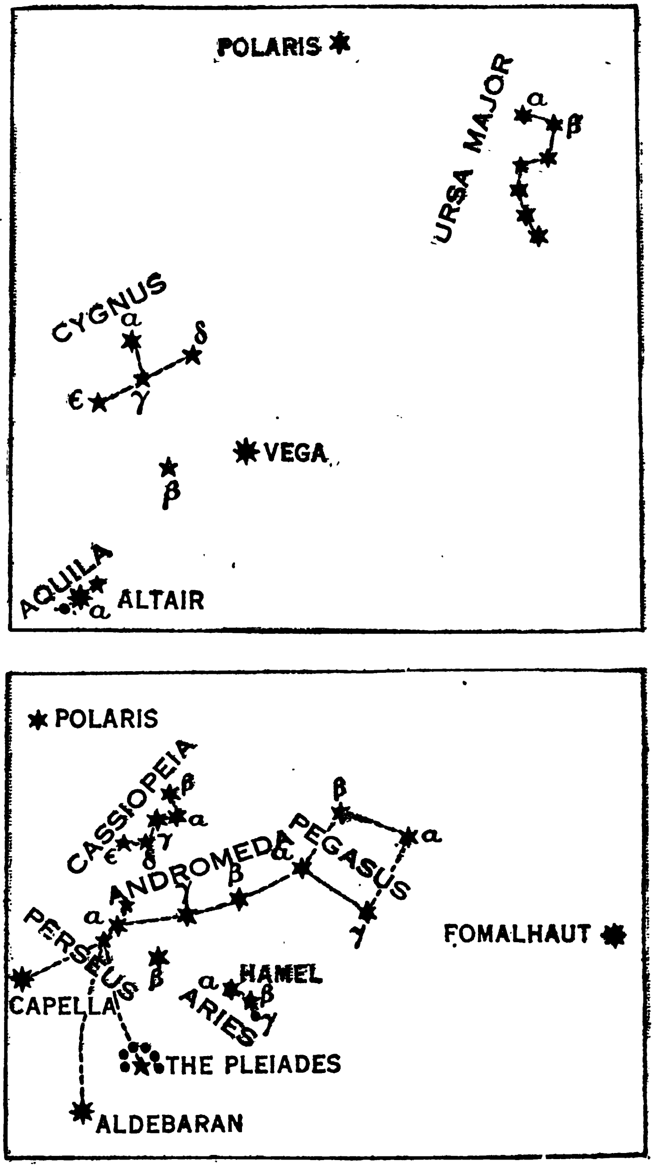


FIGURE X

75. α and β Pegasi point directly at Fomalhaut, which, therefore, also crosses the meridian at midnight on September 7. These three stars form as perfect a set of pointers to the Pole as do the "Pointers

of the Dipper and may be of great use, therefore, when the pointers, low in the heavens, are obscured by clouds.

76. *Capella, Aldebaran, Castor, and Pollux* (Fig. XI).—Aldebaran is to be found on the meridian at midnight of November 29, Capella on December 9, Castor on January 13 and Pollux on January 16. In the fall months and early winter we will find them in the east in the hours before midnight and in the west in the later winter months. A

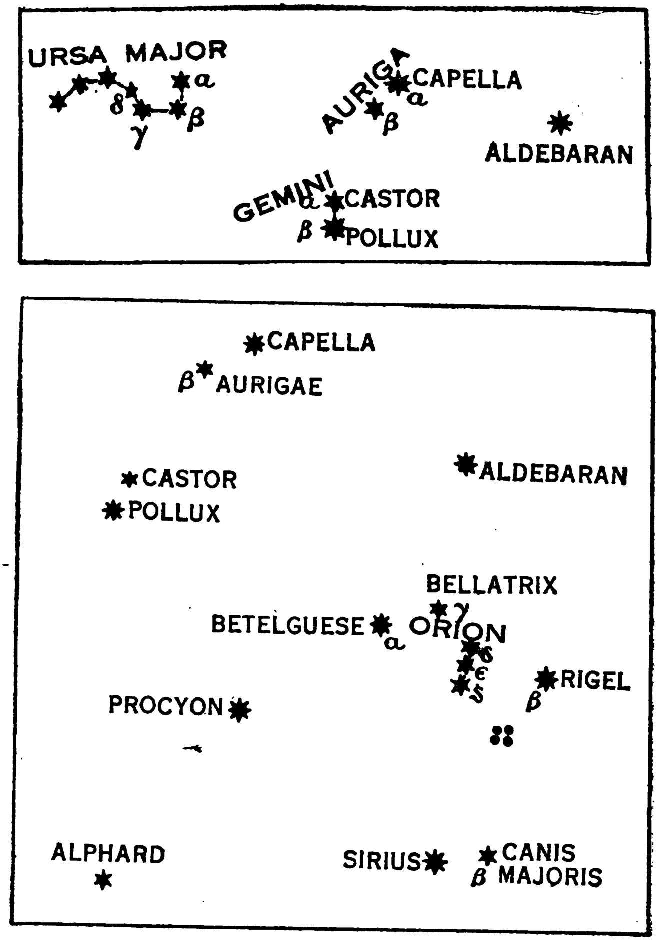


FIGURE XI

line drawn from the pole star perpendicular to the line from the "*Pointers*" passes through Capella. This line also passes directly through *Rigel,* a bright star in the constellation of Orion. These two stars, therefore, cross the meridian together and form another excellent set of north *pointers.*

The group of stars in the constellation of Orion are particularly brilliant, perhaps the most conspicuous of all star groups in the

winter heavens. Rigel, the stars in the belt of Orion, Bellatrix, and Betelguese are very near the celestial equator. Betelguese and α Aurigae transit together as do Rigel and Capella. We therefore have in the winter this double line running from the vicinity of Orion to the North Star.

Sirius, the brightest star in the heavens, is practically in line with the belt of Orion and also in line and about equidistant from the belt is Aldebaran.

77. *Date* of *midnight transit* and *declination* of *stars* shown in Figures IX–XI.

Date.	Declination.	Star.	Date.	Declination.	Star.
January 1	16.5° S..	Sirius.	July 19	8.5° N..	Altair.
January 13	....	Castor.	August 2	45.0° N..	Deneb.
January 15	5.5° N..	Procyon.	September 5	30.0° S..	Fomahaut.
January 16	28.0° N..	Pollux.	October 22	23.0° N..	Hamel.
March 7	....	Pointers.	November 29	16.0° N..	Aldebaran.
March 19	15.0° N..	Denebola.	December 9	46.0° N..	Capella.
April 12	11.0° S..	Spica.	December 9	8.0° S..	Rigel.
April 25	19.5° N..	Arcturus.	December 12	6.0° N..	Bellatrix.
July 1	39.0° N..	Vega.	December 19	7.5° N..	Betelguese.

78. The time of transit for any other night may be found approximately from the rule that the star transits four minutes earlier each successive night, about two hours each successive month, or 24 hours earlier (same date) at the end of the year.

79. The declination of a star is its angle, measured in the meridian from the equator. The latitude of a place on the earth is its angle measured in the meridian from the equator. Both are noted as north if north of equator, or south if south of equator. If the declination of a star is greater than the latitude of a place, it will pass the meridian to the north of the place. If less, it will pass the meridian to the south. Of course, if the declination is south and the latitude north, the star will always be to the south, and similarly, if the north declination of star is small and the latitude of the place high.

It is evident from the foregoing that by use of the additional sets of north-pointing stars, and with a little practice in studying the times of the year and hours of the nights when certain stars are to be found in the meridian to the north or south of us, we should seldom have any difficulty in knowing approximately at least our points of the compass.

If headquarters have, as we understand they have in France, tables from which the true azimuth of a star can be predicted for any hour of the day, and if orders based on these tables are to be issued, we must know the prominent stars. To learn them requires observation and practice, nothing else.

LESSON XI.

USE OF MAPS IN POSITION WARFARE.

80. The following condensed quotation from an article in the British Journal Royal Artillery is taken from the International Military Digest:

Now that large-scale maps, such as 1 : 20,000 or 1 : 10,000, are used extensively by batteries in the field, map reading has become an important part of an officer's

work. From the map lines of fire are obtained, aiming points picked out, the position of targets identified, and ranges obtained.

Three distinct elements must be considered with respect to any point on the ground that is to be identified on the map, viz, (1) direction, (2) distance, and (3) shape of ground or relative height. It is not safe to decide on a point which appears to fulfill two of these conditions without examining as to the third.

(1) DIRECTION.

In measuring angles a semicircular celluloid protractor (Steward's) will be found useful and fairly accurate. If several points are to be identified in a given zone, a well-defined *distant reference* point must be *selected and identified on the map*. Using the protractor, measure the angle between the reference point and *the object on the ground, then plot it off on the map*.

(2) DISTANCE.

The approximate distance may be found in two ways:

1. By estimation.
2. By *noting whether it is farther off or closer than other points easily identified* or *already known*.

As a rule the latter method will be the more satisfactory. The scale of the map should be kept clearly in mind when considering distances to objects.

(3) SHAPE OF GROUND.

This condition, though considered last, is not of least importance, since a *carefu study* of the *contours* will fix the *position of an object on the map with far* more certainty than will an estimated distance. It sometimes helps to examine all the features surrounding an object and then draw a rough plan of what is expected to be found on the map.

Often a *thick, well-defined hedge* indicates a *road*. What looks like a wood may be *only scattered trees*. A low ridge or *embankment* may conceal a *hedge*, so the *first hedge* visible beyond may be the *second hedge shown* on the map.

Watch the smoke of a *distant railway train;* it not only helps to identify the line of the *railroad*, but may be useful as a reference point for other *objects*. Finally, even the good maps may have mistakes, usually *in connection with the roads*.

PRACTICAL WORK.

81. The ability to read maps as above described can only be acquired by practice in the field. The following practical work should be executed by each member of the class in the field. If the training period permits, this character of the work should be continued until all are proficient in *reading ground* from a *distance*. A ridge in or near the training area will be selected from which a reasonably good view can be had. The members of the class with their sketching equipment and maps will be deployed along the ridge, so that as nearly as may be they have the same view. Before the lesson hour the instructor will have had placed several conspicuous objects; red or white flags will do if nothing else is at hand. A prominent object in the foreground will be designated by the instructor as the *reference point,* and each student after locating his position on the map and orienting will draw a ray to this point. He will then be required to sight and draw rays to the several flags. He will then be required to draw a small circle at the point on each ray where he believes the flag to be. He will then be required to draw a profile from his map along each of the rays. The drawing of each profile will require about 30 minutes of a beginner's time. There should not be so many required of him as to absorb the entire lesson period.

TO DRAW THE PROFILE.

82. Look along the ray very carefully and find the lowest contour which the ray crosses and also the highest. Suppose the lowest to be 520 and the highest to be 640. The highest point on your profile will then, of course, be 640 feet and the lowest 520. The difference will be 120 feet. Suppose the contour interval is 20 feet; 20 goes into 120 six times. There are, therefore, six contour intervals between the highest and lowest point. Take a sheet of paper, the long edge of which is long enough to reach from your position on map to the position of flag. With your alidade as a ruler, draw six lines parallel to the long edge of the paper at equal distances apart and from the edge. This distance may be any convenient one, but should not be less than one-half inch, if practicable. (A method of drawing the lines parallel to each other, which suggests itself, is to lay off half-inch spaces along the short edges of the paper and connect them.) Now lay the long edge of the paper on the ray. Mark the edge of the paper as elevation 520 (the lowest elevation) and the highest line as 640; mark the intermediate ones 540, 560, 580, 600 and 620. Now, beginning at your station, look along the ray very carefully, and every time the ray crosses a contour (say the first one met is the 620) run a light line up to the corresponding parallel line (620 in this case) and make a cross there. Do the same at the next contour, and continue until you reach the position of the flag. Now connect the crosses in *succession*, beginning at the first one, by straight lines. The result will be a *profile* or *vertical section* of the ground between your position and the flag's (providing you located the flag correctly). Now compare your profile with the ground between you and the flag. Does your profile help you to tell whether you located the flag correctly? Did you overestimate the distance and plat the flag's position in a valley beyond the hill where you really could not see it?

83. Draw a *straight line* on *your profile sheet* from *your position* to the *flag's position*. Does this line *clear* all *intervening points* on the profile? If not, you could not see the flag where you have plotted it. This latter little problem is what is commonly called a *visibility problem*.

84. Each sketcher will now be required to outline on his map the areas (included between the rays to two flags) which are invisible to him. To do this, he will find it desirable to draw a number of profiles. He should draw no more than absolutely necessary, and these for the sole purpose of checking a decision which he has made by comparison of the map with the ground.

LESSON XII.

VISIBILITY.

85. A problem frequently arising in map reading is that of determining *what points are visible from a given point*. A point is visible when the gradient to it, if rising, is greater, and if falling, is smaller than the gradient to any intermediate point.

For this comparison gradients are conveniently represented by the quotient of distance in feet divided by the difference of elevation in feet. The point will be visible when this quotient is smaller, if

rising, and larger if falling, than the quotient for the intermediate point. Thus, to determine whether the bridge near the Frenchman's is visible from Atchison Hill or is concealed by intermediate ground, assume the highest point of Atchison Hill to be in the center

of the 1,040 contour and to have an elevation of 1,050. The distance from this point to the bridge is 5,610 feet, fall 250 feet, quotient 22.4. The line of sight from this point to the bridge crosses the 960-foot contour on the flank of Sentinel Hill at 3,060 feet distance, fall 90 feet, quotient 34; hence bridge is not visible from Atchison Hill,

since the gradient is falling, and the nearer point has the larger quotient.

Working from the bridge the quotient for the whole distance is 22.4, as before, but the gradient is rising. The distance from the bridge to the high point is 2,550 feet, rising; difference of elevation 160 feet, quotient 16; hence, as before, the top of Atchison Hill is not visible from the bridge, since the gradient is rising, and the nearer point has the smaller quotient.

If one gradient is rising and the other falling, no computation is necessary. A point of rising gradient will hide a farther point of falling gradient, but will not be hidden by a nearer one.

86. The explanation given above may be cleared up to some as follows: The point of view on Atchison Hill is at elevation 1,050; the bridge is at elevation 800. The difference is 250 feet. If you can see the bridge from Atchison Hill your *line* of sight, as you look at the bridge, will be a straight line which slopes 250 feet in 5,610 feet, or 1 foot in 22.4 feet. Now, a straight line from Atchison Hill to the bridge crosses the 960 contour on Sentinel Hill. The point of crossing is 3,060 feet from the *point* of *view* on Atchison Hill. Your line of sight to the bridge must fall 1 foot in 22.4. Therefore, it would have fallen $\frac{3060}{22.4}$ in 3,060 feet, or 136.6 feet. One thousand and fifty feet less 136.6 feet is 913.4 feet. If you could see the bridge, your line of sight must have fallen to 913.4 feet at Sentinel Hill; but the ground there, as shown by the contour, is 960 feet high, so it interferes.

87. An instruction test as to whether an intermediate point obscures the view between two other points may be made by setting up at three points on the map pencils or other suitable objects having the corresponding elevations marked on them on a convenient assumed scale. Sight, or stretch a thread, along the pencils. If the middle mark is above the line joining the other two, each of the two extreme points is invisible from the other. If the middle point is below the line, each extreme point is visible from the other.

LESSON XIII.

RECONNAISSANCES.

88. Reconnaissance is an examination made for the purpose of gaining information relative to the terrain, the enemy's forces, or for other purposes.

The subject is of particular interest to all officers. In past wars officers have been called upon for much reconnaissance duty and such will undoubtedly be the case in any future war. In the Mexican War the close reconnaissance of the enemy's positions by officers contributed in no small measure to the success of our forces. In the Civil War the Engineer officers acting as such in the field, and Engineer officers holding volunteer commissions in command of troops, were constantly making reconnaissances and putting to use their topographical knowledge in correcting existing maps, reporting, by hasty sketches, the positions occupied by troops, and in guiding troops through close country and seeing that they were posted in accordance with the plans of the higher commanders.

89. To perform such duties well and without hesitation, the officer must possess a keen eye for *topography* and an unfailing sense of *orientation;* in short he must never *get lost,* and he should spare no pains to become thoroughly familiar with the large *drainage* and other important features of the section of country in which the *operations are being carried on.*

90. Reconnaissance should not be confused with mapping. The information obtained by a reconnaissance may be reported verbally or in writing and may or may not be illustrated by a sketch or route map. A reconnaissance is seldom made for the purpose of bringing in a sketch, but a clear sketch will save much writing and will be of value as a record and further description of the route or country covered.

91. In any reconnaissance in proximity to the enemy, the time available is a most important element. The information desired by higher authority must be obtained and to that end time should not be wasted on unimportant or irrelevant investigations. An officer on this duty must keep his mission constantly in mind and use his best judgment as to what is important and what is not.

92. The following quotations illustrate some of the various classes of reconnaissances which officers may be called upon to make:

Report of Maj. Nathaniel Milcher, Corps of Engineers, United States Army, Acting Chief Engineer, Headquarters, Army of the Potomac, Engineer Department, 1864:

* * * * * * *

Although the enemy has no doubt suffered at times from want of accurate maps, still he has at all times possessed a superior knowledge of the country, and could always obtain reliable guides from among its inhabitants, thus affording him a very great advantage over his adversary. In order to be able to cope with him with anything like equal advantage, it soon became apparent that the difficulties to which reference has been made would not only have to be overcome by gathering material with the onward march of the army, but that the desired information would have to be obtained in anticipation of any move. To accomplish this the officers and assistants of my party were kept constantly occupied both day and night; they were not only called upon to prepare the much-needed maps with the detailed corrections, but also in the entire absence of reliable guides to act as such to the different columns, either as they moved along their respective routes of march or while maneuvering for favorable positions previous to an attack.

On the morning of the 8th, some severe skirmishing commenced between the advance of the Fifth Corps and the enemy, showing that the latter was falling back from the wilderness toward Spotsylvania Court House. At break of day I was directed to make a reconnaissance of the country along the Brock Road and parallel to the Po River, to select **a good position for the Second Corps to take up** in the event of the **enemy attempting to strike our flank.**

Early on the 10th the Second Corps was advanced across the Po by ponton bridge. Subsequently, by order of the commanding general, **I guided Gibbon's and Birney's divisions back again across the river and placed them in position to the rear and right of the Fifth Corps,** where they were massed to make a combined assault. Lieut. Mackenzie was on the same day engaged **on a reconnaissance to the front of the Sixth Corps,** and, in company with General Russell, **selected the point of attack** so successfully made by Upton's brigade of that corps.

On the 1st of June * * *, during the day, accompanied by Capt. Gillespie, who had joined the army a few days previous, and several assistants, I directed the examination of the country to the southeast of the Old Church Tavern **for the purpose of finding several parallel roads over which to move simultaneously different columns.**

On the morning of the 10th * * *, Capt. Mendell, accompanied by Lieut. Howell, made a reconnaissance to Windsor Shades, on the Chickahominy, to ascertain the practicability of crossing at that point the supply train, but reported unfavorably.

On the 24th, accompanied by Capt. Mendell and Lieut. Howell, I made a reconnaissance of the country between the Avery House and the Blackwater Swamp, for the purpose of selecting a line * * * the crossings of the swamp were also carefully searched, and its character examined in regard to forming an obstacle to the passage of artillery and infantry. * * * On the 29th the Appomattox was also examined in reference to the facilities for bridging it.

Gen. Sheridan's expedition toward Gordonsville returned on the 30th, and the assistants who accompanied it brought back most valuable topographical information, among other interesting matter a survey of the enemy's works at Spotsylvania Court House. This latter enables me to furnish in full, and with accuracy, the battle field map of that locality.

93. From the above, it is seen that reconnaissance made by officers serving with troops in the field fall naturally under the following headings:

(*a*) Reconnaissance of a river crossing.
(*b*) Reconnaissance of a defensive position to be occupied by our own troops.
(*c*) Reconnaissance of a position occupied by the enemy with a view to finding the best point for an attack.
(*d*) Reconnaissance of routes of march.

LESSON XIV.

RIVER CROSSING.

94. If a *crossing* is to be made in the face of the enemy, the location selected must fit the *tactical situation;* that condition being complied with, choose the *location* which will require the *least labor* and *material* to render it practicable.

Fords should not be more than 4½ feet for Cavalry, 3½ feet for Infantry, and 2½ feet for guns and wagons. The nature of the stream bottom is most important. It should give *good footing and should not scour under the action of wheels and hoofs.* Almost any ford except one with rock bottom will scour under the continued passage of Artillery trains. The *velocity* of the current and ease of approach and exit must be considered. If work is necessary to prepare the immediate approaches, or roads connecting with the nearest main roads, an *estimate* of the men, tools, and time should be made.

PONTON BRIDGES.

95. Other things being equal, a point where the stream is narrow should be selected; this facilitates speed of construction. *Low, firm banks and easy approaches* are *very important.* A marshy approach, or a steep bank 10 feet or more in height, may require more time in preparation than the construction of a bridge several hundred feet long. In the Civil War the *principal work in the connection with ponton bridge crossings* was *the preparation of corduroyed approaches.* If the enemy is opposing the passage from the opposite

bank tactical conditions are paramount. A point should be selected where the *approach* to the bank *is concealed by trees* or otherwise, where the *bridge will be screened from the enemy's artillery observers,* and if possible, where there is little cover for the enemy's infantry on the far bank. The lower reach or mouth of a tributary often fulfills these conditions. The bank of departure should preferably be the higher of the two, and the site should be such as to permit the concentration of artillery and rifle fire on all points from which the enemy might oppose the passage. The *concave side* of the bend of an alluvial stream often fulfills the above requirements for bank departure, and has the added advantage that *most of the approach work* will *be on that* (the high) *side and can be carried on while the bridge* is being constructed. If the farther bank is occupied by the enemy and his small-arms fire can not be silenced by artillery and small-arm fire, an attack must be made before the construction of the bridge can be begun. This should be delivered by Infantry ferried across in ponton boats. A sheltered spot for launching and loading the boats is most desirable, but the operation may be effected with fair rapidity *on the open bank, particularly under cover of darkness.*

IMPROVISED BRIDGES.

96. By improvised bridges are meant those which are constructed of material collected from the vicinity of the site. On account of the *length of time required* to *collect the material* and to improvise a suitable bridge out of the *miscellany,* such bridges will seldom be built for the *advance troops.*

These troops must, when time is pressing, get across the stream by fording or by ferrying, etc. The reconnaissance officer must, therefore, in every case when time is pressing, and ponton equipment is not at hand, *search for available fords and for boats to use in ferrying.* There should be no delay in transmitting information as to fords or ferries to the troops.

The crossing of large bodies of troops either by fording or ferrying is, however, very unsatisfactory, and the reconnaissance officer should, after he has reported on fords and ferries, look up material and site for an improvised bridge, to be constructed by the engineers while the advance troops are crossing.

97. The *reconnaissance,* therefore, will have *two phases.* The first, a search for means available for an immediate crossing. *The report on this phase will be made at once* and it may be necessary for the reconnaissance officer to guide the troops to the ford or to supervise the passage by boats. This part of his work completed, he continues the second phase of his reconnaissance. *He must now search out and note where the material can be procured* and by what routes and transportation it can be moved to the site selected. *He must then examine the site sufficiently to be able to give to the constructing officer* the width of the stream and the character of the bottom.

To meet emergencies such as the above it is of prime importance to have ponton material always with the advance troops. With a large command improvised bridges will usually be built for the purpose of releasing the ponton material so that the bridge train can proceed in advance.

LESSON XV.

RECONNAISSANCE OF ROUTES OF MARCH.

98. If time is pressing, such a reconnaissance may consist simply of riding the road, keeping oriented by the sun or compass, and making notes of important features either on paper or mentally. If there is sufficient time for better work, a sketch of the route traveled should be made. This may be done in any one of the ways described hereafter, which best suits the conditions and the personal preference of the officer making the reconnaissance.

99. A good sketch will show the road followed and the important **topographical** features as far to either side as they can be drawn from actual observations. By important features is meant the important drainage lines and the hill forms as controlled by those lines. It must be kept in mind that a road has absolutely no influence on topography. *The topography was there first and the road was put in on top of it afterwards.* It is a common mistake to assume that, because in carrying approximate elevations, readings are better taken on slopes **along a road**, the contours or form lines cross the road at right angles. The result is a sketch showing a succession of ground waves, ridges, and valleys, all perpendicular to the road, which is of course entirely worthless so far as giving any real information of the country is concerned.

100. There are but two things absolutely essential in the making of a good road sketch, first, **a good traverse**, and, second, the **location of the drainage system** in its proper relation to that traverse. With this control approximate contours can be drawn by anyone having a knowledge of topographical principles. Ground forms so obtained will never be misleading, and if drawn by a skillful topographer will give valuable information. Time should not be spent on plotting unimportant details, for in addition to wasting time the really valuable parts of the sketch are thereby obscured. *Prominent buildings and farmhouses are of value in assisting anyone using the sketch* in locating himself thereon. Wooded areas and orchards are for the same reason important, and may be more so for tactical reasons. It should not be expected that a route sketch will show every fence, ditch, or bit of cover that might hide a patrol, and a striving after such detail only befogs the **real issue, that of finding where the road goes and how**; and **in general what the country adjacent to it is like.**

101. In road *sketching* with the issue table, the tripod as a rule is not used, and to get good orientation it will usually be necessary to dismount, as the average horse will not stand still while an observation is being made. Points off the road can be located by intersection or by direction and estimated distance. Distances are plotted along the road by means of a *time scale,* if using a *horse* or a *scale reading* to the *least reading* of *your speedometer* if using an automobile. The speedometer should first be tested by rating over a known or measured course, and the horse should be timed at a walk, trot, and gallop over a measured course.

102. *To make a time scale,* having rated the horse over a known course: Suppose the scale of your map is 1 : 10,000. Suppose your

course is 8,280 feet long and your horse takes 9 minutes to travel the 8,280 feet. He, therefore, travels $\frac{8{,}280}{9}$ or 920 feet in one minute. Reduce this to inches by multiplying by 12 and we have 11,040 inches. The distance on the map to represent this will be $\frac{11{,}040}{10{,}000}$ or 1.1 inches on the map will represent a *minute* of the horse's gait. Lay off 1.1 inches several times on a suitable piece of paper. Each division will represent one minute of travel at that gait.

103. *To make a scale for speedometer.*—These instruments usually read to tenths of a mile. They may not be accurate. Run over a *measured course* at least 10 miles long. Suppose your speedometer shows 9.2 miles; the tenth-mile register is, therefore, inaccurate. Lay off the proper distance 10 miles to scale and divide it into 92 equal parts. Each one of these parts will represent the distance traveled for each tenth register of your speedometer. See paragraph 217 for explanation of method of dividing a line into a given number of equal parts.

LESSON XVI.

COMBINED ROAD SKETCHING.

104. As soon as the individual sketchers have shown a proficiency in road reconnaissances, at least one exercise should be had in combined work. In combined road sketching the work of individual sketchers is in no way different from that heretofore described, but strict precautions must be taken to identify the proper location of each sketcher's work in the combined sketch. To accomplish this, it is absolutely necessary to mark every crossroad by a distinctive card, or otherwise, and this distinctive mark must appear on each sketch which starts or terminates at, or includes that point.

105. A method which has been found to work well in peace times is as follows: Suppose it is necessary to reconnoiter three roads leading toward the front and generally parallel, also all connecting roads and all roads for a distance of one-half mile leading to the right from the right-flank road and to the left from the left-flank road. The problem is primarily one of organization. Three parties should be organized as follows:

Party A:	
For right-flank road—	
1 director.	
1 principal sketcher.	
Assistant sketchers, say 3	5
Party B:	
For center road—	
1 director.	
1 principal sketcher.	
Assistant sketchers, say 6	8
Party C:	
For left-flank road—	
1 director.	
1 principal sketcher.	
Assistant sketchers, say 3	5
	18

106. Each director should be supplied with an aneroid barometer and all three barometers should be set at the same reading on starting out.

The duty of the director of party A is to give general instructions to his sketchers and to then ride his road, putting up at each cross-road or side road a card having marked thereon an arrow showing the direction to be followed by his principal sketcher, the letter of his party, the number of the cross or side road counting from the start, and the aneroid elevation, as A_1, 850; A_2, 980, etc.

The duty of the principal sktecher of party A is to sketch the right-flank road indicated by the cards and to put on his sketch at each cross or side road intersection the letter and number of the card found at that point. Assistant sketchers take the side roads in succession.

The duty of each assistant sketcher is to mark the starting point of his sketch with the letter and number of the card found at that point, then to sketch his road to the flank the distance ordered. He should place his initials on the card before leaving. On completing his work, unless otherwise ordered, he returns to the right-flank road and sketches the first road he comes to which has not been initialed by another assistant sketcher.

The duties of the director of party B are similar to those of the director of party A and those of the principal sketcher of party B are similar to those of party A. The assistant sketchers of party B are sent off to the right and left of the center road as cross or side roads are encountered. As above, each must place at the starting point of his sketch the letter and number of the card found at that point. He must follow through and sketch his road until he finds a card on the right or left flank road and he must place the letter and number of this card at the end of his sketch. If his road does not run through, he must indicate on his sketch that it does not. The duties of the director of party C are similar to those of the director of party A. The duties of the principal sketcher and assistants are similar to those of party A, but opposite hand.

107. If all the sketches are marked as indicated there will be no trouble in combining them, although, of course, they will not fit exactly. Copies may be made by blue printing from each sketch separately and combining the prints, or by tracing the combined sketch and printing from the tracing. Sketchers must be instructed as to time and place for turning in their work.

LESSON XVII.

COMBINED AREA SKETCH.

108. If any map is available the area to be reconnoitered should be outlined on it and subdivided into as many tasks as there are sketchers, the parts being apportioned according to the time necessary to complete them so that all sketchers will be in at about the same time. Each of the parts is assigned to a sketcher, with definite instructions as to the amount and class of work to be done, the scale to be used, which should be the same for all, and the place and time at which the sketch must be turned in.

If the available map will give fairly good control it should be enlarged to the scale required for field work and a section of the map given to each sketcher with his instructions.

The boundaries between sketchers must be physical features which are shown on the available map, such as roads, trails, streams, etc. These boundaries should appear on both adjacent sections of the map as handed to the sketchers. If a sufficient number of copies of the available or base map are not at hand to cut into sections with overlaps, the necessary boundaries or the control for each sketcher may be traced from the base map.

In the absence of an available map if a combined road sketch be made and satisfactorily combined it will furnish a reasonable satisfactory base map for an area.

LESSON XVIII.

BRIDGES.

109. In reconnaissances for routes of march it is essential that special attention be called to all bridges which are in bad condition or too light for ordinary army loads. If guns heavier than the 4.7-inch rifle or the 6-inch howitzer, heavy trucks, etc., are to use the bridges it will be desirable to have all bridges examined with care, and reference should be made to the Engineer Field Manual, chapter on "Bridges."

Every officer, however, should be able to determine quickly when a bridge is undoubtedly safe for ordinary army loads and when it is undoubtedly unsafe. This he will determine ordinarily by mental comparison with other bridges *known* to be safe or unsafe. To develop the faculty for *mental* comparison he must have practice in examining bridges. The following *thumb rules* will aid him:

PLANKING.

110. For ordinary army loads the planking should be as thick in inches as the stringers are apart in feet. For 4.7-inch guns or 6-inch howitzers it would be well to lay *wheel tracks* whenever the planking is less than 3 inches thick.

STRINGERS.

111. Let b represent the breadth of stringer in inches, d the depth in inches, and L its length in feet between supports. The *index*, as it may be called, is equal to the breadth in inches multiplied by the square of the depth. Therefore, to find the *index*, measure the breadth and depth of a stringer in inches and multiply $b \times d \times d$.

Now measure the distance in feet between the *supports* of the *stringers*. In truss bridges this is not the distance between the piers. It is the distance between the tranverse beams (usually steel) which carry the roadway. Count the stringers. There must be four or more or the thumb rule will not apply. Divide the *index* by the *length* in feet between supports. If the *quotient* is *15* or over, the stringers are safe for loaded *escort wagons* or *3-inch guns*. If the quotient is *30* or over, the *stringers* are *safe* for *4.7*-inch guns. If the length of stringers between supports is not over 15 feet (that is not over twice the distance between axles of the carriages) the

quotient may be reduced to two-thirds of the values given and the stringers will still be safe.

112. *The stringers* may be steel I-beams. If they are, measure their *depth*. Square this for the I-beam index. Divide this index by the length between supports in feet. If the quotient is 1½ or more the stringers are safe for 3-inch guns. If the quotient is 3 or more, the stringers are safe for 4.7-inch guns. If the length of stringers between supports is not over 15 feet, these quotients may be reduced to *1* and *2*, respectively.

ROADWAY BEARERS.

113. The *index* of wooden roadway bearers divided by the length of the *bearer* between *its* supports should be 50 or over; if steel, 5 or over. This rule is based on a spacing between road bearer of 15 feet. If the spacing is 20 feet, the quotients should be doubled; between 15 and 20 feet, interpolate.

It will be understood that the above thumb rules are given as an aid to the nontechnical officer. They are not *construction rules.*

LESSON XIX.

RECONNAISSANCES OF POSITIONS.

114. It is believed that the reconnaissance of a position to be taken up by our own forces has no place in this Manual. *Tactical considerations govern.* Something has been said of the high-class work required in securing a *base skeleton* of an enemy position in the "position warfare" of to-day. The following quotation is given as a suggestion of a possible method of securing such a skeleton.

115. **Operations in front of Petersburg.**—Extract from report of Maj. Nathaniel Michler, Corps of Engineers, United States Army, 1864:

> On the 9th of July, 1864, orders were issued by the commanding general that "the operations of this army against the intrenched position of the enemy defending Petersburg will be by regular approaches on the fronts opposed to Gen. Burnside's and Gen. Warren's corps," and on the following day a plan of conducting the siege was submitted.
>
> On learning the plan adopted, I directed my principal assistant, Maj. John E. Weyss, to commence on the 9th an exact triangulation of the front of Petersburg, locating our own line of work as well as that of the enemy, and to take the immediate charge of the surveying party. My assistants, Messrs. Theilkuhl, Schumann, and Jacobsen aided him. The work was extended from the south of the Jerusalem plank road as far north as City Point. By this triangulation, performed under the fire of the enemy's batteries and sharpshooters, the different spires and certain prominent buildings in Petersburg were accurately located, and having been kindly furnished by Prof. Bachs, Superintendent of the United States Coast Survey, with a copy of the beautiful map of that city and the Appomatox River prepared a few years ago in his department, I was able to combine the two, and thereby obtain an exact connected map of the locality of our siege operations, covering the whole ground occupied by both armies.

116. Large scale surveys, such as are described above and as have undoubtedly been made of the various fronts in Europe, will ordinarily be executed by expert surveyors attached to army headquarters; but triangulation of a reasonable degree of accuracy and topographical surveys controlled by such character of triangulation or by transit traverses will from time to time be required of the divisional Engineers. For this reason PART II has been added to this manual on the use and adjustments of the various instruments

used in surveying for the measurement of angles, distance, and elevation. The following plan of procedure is suggested for a topographical survey of an area too large to permit of the use of reconnaissance methods and yet not large enough to justify geodetic methods. The time allotted to this training course and the personnel of the class will determine whether instruction can be given in this part of the manual.

LESSON XX.

TOPOGRAPHICAL SURVEY OF AN AREA APPROXIMATELY 100 SQUARE MILES.

117. In this work, as indeed in all map work or sketching, it "must be accepted as an axiom that however large or however small may be the area to be surveyed, it must be treated as a whole, and that all over the area a *number* of *carefully determined points* must be fixed," and these points adjusted among themselves to form an accurate framework on which the less accurate work may be hung.

The accuracy of the entire map will depend upon the accuracy of the framework or control. The framework may be located by *triangulation* or by *transit traverses*. A combination of the two will be the ordinary rule.

TRIANGULATION.

118. Paragraphs 187 to 189 give the methods to be followed in measuring the *base line*. It should be remembered that an error in the base line will be reproduced in the triangulation. Thus, if the base measurement is $\frac{1}{1000}$ larger than it should be and the farthest point of the triangulation is actually 10 miles from the base, the triangulation will place the point at a distance of $\frac{1}{100}$ mile, about 50 feet too far. This would not be serious if the work is to be carried no further.

ANGLE MEASUREMENT.

119. If *possible*, all of the angles of each triangle should be measured. The procedure at each station is as follows: Level carefully with the telescope level and see that the plate levels and other adjustments are satisfactory. For each principal angle: Set the A vernier to read zero and, with the telescope direct, set with the lower motion carefully on the left-hand station. Read and record both verniers. Unclamp above and set on the right-hand station. Read and record the A vernier for the *approximate angle*. Find the number of whole times 60 will go into this approximate angle and call this M. Then—

Unclamp below and set on the left-hand station Unclamp above and set on the right-hand station	Second repetition.
Unclamp below and set on the left-hand station Unclamp above and set on the right-hand station	Third repetition.

Plunge the instrument, and without disturbing the vernier setting,

Unclamp below and set on the left-hand station Unclamp above and set on the right-hand station	Fourth repetition.
Unclamp below and set on the left-hand station Unclamp above and set on the right-hand station	Fifth repetition.
Unclamp below and set on the left-hand station Unclamp above and set on the right-hand station	Sixth repetition.

Read and record both verniers. The mean of the "seconds" of the two verniers will be taken with the degrees and minutes of the A

vernier as the vernier reading. To this must be added 360° multiplied by M as above determined. This large angle is then divided by 6. This is the value of the angle for one set of readings. A second set should now be taken. In the second set the vernier at the start should be set at 35°. If the values from the observed sets do not check out, the observations are immediately repeated, if they do take the mean.

120. In reading vertical angles, level the instrument with the telescope level. Measure and record the HI. Read, direct, the vertical angles. Plunge the telescope and reread the vertical angles. Relevel the instrument and repeat the direct and reverse observations. The recorder notes the point observed and the vertical angle for each station, landmark, or flag.

121. After the angles which pertain to the triangulation system have been read and recorded as above described, a series of "pointings" are taken to all spires and other prominent landmarks in view, record of each object "pointed" being kept in such a manner that there will be no difficulty in identifying it later, when "pointed" from another station. These pointings are taken as follows:

Set A vernier at zero, and with the telescope direct, set carefully with the lower motion on any *principal station*. Then, unclamping above, read the angles to the successive landmarks in turn around the horizon, closing and reading in the original zero line. Set B vernier at zero and, with the telescope reversed, again set by lower motion on some principal station. Unclamp above and read landmarks as before, closing on the zero line. In case either reading on the closing line or direct and reverse reading on any landmark do not check within one minute, repeat the entire operation.

THE STATION ADJUSTMENT.

121. After all the *main angles* have been satisfactorily observed, closing the horizon, their sum should equal 360°. The difference between 360° and the sum will be divided equally and applied as a correction to each angle, so that the resultant sum is 360°. This adjustment is made in the notebook, and the final values checked.

REDUCTION TO CENTER.

122. In triangulation of the character being described, it frequently happens that angles are taken to some point, such as a tree, where it is impossible to erect the transit. In such a case the instrument is set up at the nearest convenient point, called a *satellite station*, at a short distance from the tree, and the angles taken from this station can be afterwards reduced to the true values at the object itself.

From the satellite station which we will call S, the round of angles will be read as usual, taking care to include the tree which we will call T. Measure the distance from S to T carefully. Let A and B represent two stations from which T has been observed, A being on the same "hand" of B as is S from T, in this case the left. We measured the angle ASB. The angle desired is ATB.

$$(a)\quad ATB = ASB + SAT - SBT.$$

To find SAT and SBT we need to know the length of AT and BT approximately. To determine them approximately, we can plot the triangle ASB, lay off ST in direction and distance as observed and measure AT and BT. This should be done at a large scale. Now—

$$(b)\ \text{Sin SAT} = \text{Sin TSA}\frac{\text{ST}}{\text{AT}}\ \text{and}$$

$$(c)\ \text{Sin SBT} = \text{Sin TSB}\ \frac{\text{ST}}{\text{BT}}.$$

From (*b*) find Sin SAT and from it the angle SAT.

From (*c*) find SBT, substitute these values in (*a*), and we find *ATB*. (See fig. 14.)

ADJUSTMENT OF ANGLES OF TRIANGLES.

123. In securing the triangulation control for an area of 100 square miles or so, the purpose of which is primarily the construction of a map on a reasonably small scale, the computations necessary for the adjustment of quadrilaterals are not justified. Each triangle, therefore, will be adjusted as a single figure as completed and the computed sides considered as correct, unless, of course, an actual mistake is discovered. It should be realized, therefore, that the fewer of these main control triangles we use, the better for the work. If the country is suited to triangulation, we may wish to have a number of small triangles to control the actual taking of the topography, but these *should not* be made a part of the main triangulation system. They should be adjusted *to it,* not with it. Except for station adjustment of angles, which is of course desirable, nothing should enter into the adjustment of any of the main triangles but the angles at the verticals of *each triangle.*

124. It is evident, therefore, that if the triangulation net is extended to the sides as well as forward that each vertex located to the side must depend for its adjustment upon the side used for a base for its determination and no interadjustment between adjoining side triangles must be attempted. Do the work carefully and you will have confidence in the resulting system of control.

124. The angles of any one of the main triangles are to be adjusted as follows: Add them together. The difference between the sum and 180° is the error. (This should not be greater than 30 seconds and will be less if care has been taken to use rigid points to sight on and light conditions have been favorable.) Distribute the error equally (not proportionately) among the three angles so that their sum will be 180°.

AZIMUTH.

125. The initial meridian of the triangulation should be a true north and south line. Azimuths in this work are reckoned from the initial meridian or lines drawn parallel to it; from the south point, in the direction S–W–N–E and from 0° to 360°.

COMPUTATION OF SIDES.

125. The base line is the only side of the triangle whose length is known. The other sides are to be found from the law of sines, that is—

$$\frac{\text{Sin A}}{\text{Sin B}}=\frac{a}{b} \qquad \frac{a \text{ Sin B}}{\text{Sin A}}=b$$

$$\frac{\text{Sin A}}{\text{Sin C}}=\frac{a}{c} \qquad \frac{a \text{ Sin C}}{\text{Sin A}}=c.$$

Assuming *a* to be the *base* and the angles A, B, and C to have been measured, the calculations are arranged as follows:

(1) log a (1400.74) =3.1463575.
(2) colog Sin A (57° 42′ 16″) =0.0729874.
(3) log Sin B (61° 17′ 53″) =9.9430639.
(4) log Sin C (60° 59′ 51″) =9.9418088.
Sum of (1) (2) (3) =log b =3.1624088.
Sum of (1) (2) (4) =log c =3.1611537.

COMPUTING THE COORDINATES.

127. The next step in the computations is to put the results in a form which permits accurate plotting and which enables a record to be *kept* of the position of each trigonometrical point. The system used will in this work be rectangular coordinates.

128. If practicable, the origin of coordinates should be so located that the entire area to be surveyed will be north and east of it. The coordinates of all points of the area will then be plus. This is to be desired.

129. It is to be understood that in rectangular coordinates the *initial meridian*, that is, the north and south line through the origin, is *the only true north and south line of the survey.* All azimuths are referred to it and the convergence of meridians is neglected. This is allowable in small areas. Through the origin of coordinates, two lines are imagined, one line north and south, which is the initial meridian, the other perpendicular to that line at the origin. The location with respect to the origin of any point is established when its *departure* (perpendicular distance from the initial meridian) and its latitude (perpendicular distance from the east and west line axis) are *known.*

130. The sign of departure when east of the initial meridian is plus. The sign of a latitude when north of the origin of coordinates is plus. It is usual to speak of the initial meridian as the Y axis, and the other as the X axis. Differences in latitudes are, therefore, spoken of as differences in Y, and differences in departure as differences in X.

131. If the coordinates of the point of origin are *known* or assumed, the coordinates of any other point which is tied to it by angle and distance may be found by adding, algebraically, the difference in X and the difference in Y to the coordinates of the point of origin. The difference in coordinates between the origin and the other point will be obtained as follows:

Difference in X = distance to point x sin azimuth.
Difference in Y = distance to point x cos azimuth.

The coordinates of any new point may similarly be determined from those of any other point whose coordinates are known and to which the unknown point is *tied* by angle and *direction*.

LESSON XXI.

TRAVERSE CONTROL.

132. The purpose of the higher or triangulation control just described is to establish a *few points* in the area with a considerable degree of precision upon which to hang the control of lower order. If the country does not lend itself to triangulation, these few points may be established by a traverse control.

133. If a railroad or reasonably level road runs through the area, it is quite possible to traverse it and measure it by stadia with an accuracy of 1 in 500. If the distance across the area is 10 miles (52,800 feet) the total error in the line would be about 100 feet, probably quite as good a "fixation" as would be made by rapid triangulation. There is a disadvantage, however, in the single-line traverse for main control in that there is no check on mistakes in distance.

134. If the triangulation system has established a known point on which to close the traverse, well and good; but if there have been no points established by triangulation, the main traverse must be run as a loop, closing on itself or be remeasured as is a *base* line. The following procedure is preferred for this class of traverse: Through the origin a true north-and-south line will have been laid out. The first set up is over the point of origin, and instrument is set to read *zero* pointing to a pin in the meridian south of the origin. This pin is the orientation station for the first set up.

135. **At the first station.**—Party arrives at the first known position.

I. Transit is set up carefully over the station, with tripod screws unclamped. Level approximately; center the instrument accurately, with the plumb-bob just above the mark. Clamp the tripod legs.

II. Level with telescope level. Lower the needle. Measure and record height of instrument.

III. Set the "A" vernier to read the azimuth to the orientation point. If the A and B verniers do not agree in minutes, so set "A" that the mean of the minutes on A and B will be the correct azimuth.

IV. With the lower motion, sight accurately on the orienting station, clamping securely below.

V. Read and record magnetic bearing. Recorder makes note of declination. (Even with local attraction, the needle will check the front with the back sights from the same station.)

VI. Unclamp above and take side shots, if any, reading (1) stadia, (2) control vernier "A," (3) vertical angle. In taking these side shots, clamp the alidade lightly.

VII. After the last side shot, sight again on the orienting station, checking the vernier reading. Plate levels are observed to see that the instrument has not gotten out of level.

VIII. The front rodman, having chosen the new station with respect to the conditions of having a good line further on, topographic control and proper distance from the instrument, is signaled "up." He presents the edge of his rod vertically over the new mark.

IX. At the instrument sight carefully with the upper motion on the new station, clamping firmly. Signal the front rodman, who then presents the flat stadia vertically over the mark. The rod rests on the mark, not on the ground.

X. Read and record (1) the stadia intercept; then set the horizontal wire on the stadia number corresponding to the HI. Glance at the plate levels to see if the instrument is still level. Read and record in order (2) control vernier "A," (3) vertical angle, (4) check vernier "B," (5) magnetic bearing (needle). The recorder repeats all readings as given, notifying the observer if the vernier readings or the magnetic reading do not check. (See par. 139.)

XI. Signal down the front rodman. Raise and clamp the needle. Put telescope in carrying position. Loosen the lower motion. Observer with instrument and recorder move to forward station. Rear rodman stays at the station.

136. **At the second station.**—

I. Set up accurately as before, level with telescope level. Unclamp the needle. Measure and record the HI.

II. Check the reading of the "B" vernier, which at this station is control. If there has been a slip, reset the "B" vernier at the same reading the "B" vernier had on the last foresight. Signal "up" the rear rodman to present the edge of his rod vertically over the old station.

III. With the upper motion still clamped and the telescope direct (not "plunged") sight carefully with the lower motion on the rear station, clamping firmly.

IV. Signal the rear rodman to present the flat of his rod vertically over the mark. Read and record the stadia intercepts, the recorder noting the check with the former fore shot.

V. Set the horizontal wire on the stadia number corresponding to the present HI, glancing at the plate levels.

VI. Read and record in order (2) control vernier "B," (3) vertical angle, (4) check vernier "A," and (5) magnetic bearing. Recorder notes if the verniers check, if the vertical angle checks with former front shot. If the vertical angle does not check, remeasure the HI, relevel with the telescope level, check the vertical circle adjustment and reread the vertical angle. Take this value as final, the recorder making special remarks to that effect in the notes.

VII. Signal down the rear rodman. Read the necessary side shots as before, clamping the alidade (upper motion) lightly. The first side shot should be well-defined landmark, preferably at least 800 feet distant as an orientation check. On this shot read and record both verniers and the magnetic bearing.

VIII. After taking the last side shot, call up the front rodman who has chosen the new forward station and who presents his rod as before.

IX. Check the azimuth on the orienting point (first side shot), noting that the instrument is still level.

X. Sight carefully on the forward station, clamping the alidade (upper motion) firmly. Then read and record as before (1) stadia, (2) control vernier ("B" for this station), (3) vertical angle, (4) check vernier "A," (5) magnetic bearing. The "B" vernier has remained control for this station.

XI. Clamp the needle, loosen the lower motion, put the instrument in carrying position and move forward again.

137. **At succeeding stations.**—Proceed as above. "A" and "B" alternate as the control vernier. In this way, the azimuth is carried forward without introducing errors of collimation, and errors due to eccentricity, inclination of the horizontal axis and vertical circle adjustment, are automatically balanced and corrected.

138. If, on closing on a triangulation station or closing the traverse on point of beginning, the azimuth is not out sufficiently to indicate a mistake, the error in azimuth will be divided by the number of stations and distributed, the changes being noted in ink in the note book. It will be noted that the control vernier gives the degrees of azimuth. The minutes taken will be the mean of the minutes of the two verniers.

139. **Method of recording traverse notes.**

L. A. traverse from ◬ Asan to ◬ Agana.

Station.		Stadia.	Azimuth.		V. A.	Diff. El.
From—	To—	H. Dist.	Control.	Check.		Elev.
			° ′	° ′	° ′ ″	
	⊙5	525	152 06	332 06	−0 01	
⊙4	⊙3	820	320 41	140 21	−0 41	
					+0 40 30	
	⊙4	820	140 41	320 41	+0 40	
⊙3	⊙2	935	314 57	134 57	−0 06	
		935.5	....	....	+0 06 30	
	⊙3	936	134 57	134 57	+0 07	
⊙LAZ	⊙1	347	310 28	130 28	+0 05	
		348	....	....	−0 04 30	
	⊙LAZ	349	130 28	310 28	−0 04	
⊙LAI	◬Asan	135	45 10	225 10	+3 25	
	Mean	134.5	....	....	0 24 30	
	LAI	134	225 10	45 10	−3 24	
◬Asan—	◬Chachao	....	179 32	259 32	+1 11	

140. The azimuth error distributed, the differences in X and Y are computed as stated in paragraph 131.

Care must be taken to give the computed latitudes and departures their proper signs. With azimuths reckoned from the south point as zero and in the direction of the movement of the hands of a clock;

For azimuth between 0 and 90, latitudes are negative and departures are negative.

For azimuth between 90 and 180, latitudes are positive and departures are negative.

For azimuth between 180 and 270, latitudes are positive and departures are positive.

For azimuth between 270 and 360, latitudes are negative and departures are positive.

141. If the azimuths and lengths of courses had been exactly determined in the field and no error has been made in computation, the survey would close, that is, the sum of all the plus latitudes (northings) would equal the sum of all the minus latitudes (southings) and the sum of all the "eastings" would equal the sum of all the "westings." Such exactness is, of course, not attainable and the sum of all the "northings" will not equal the sum of all the "southings," nor the "eastings," the "westings."

142. The total error in latitude is the difference between the sum of the northings and the sum of the southings. Similarly the total error in departure is the difference between the sum of the eastings and the sum of the westings.

143. As the errors in latitude and departure are those due to distance alone (the error in azimuth having been distributed before computations), the corrections should be applied according to the lengths of the courses. The rule is: The correction to be applied to the latitude of any course is to the total amount of the error in latitude as the latitude of that course is to the sum of **all the latitudes** (without regard to algebraic signs). A similar rule determines the correction in departure for each course. The corrections to latitudes and departures of each of the courses having been applied, the total latitudes and departures (**coordinates**) of a station are found by adding algebraically to the latitude or departure of the initial point of the traverse, the algebraic sum of all the latitudes or departures of the preceding courses. These coordinates are used for the plotting of the stations.

144. **The main control traverses** having been run, computed, and adjusted as above described, the stations of the traverse are plotted in ink, station numbers and elevations being noted in pencil. This done, a careful tracing of all the data on the projection sheet is made on vellum, cut to a size to fit the engineer sketching board. Each topographer is given one or more of these field sheets to fill in. When practicable, the areas assigned to a topographer will be those over which he and his party have been engaged in running the main traverses. Boundaries of these field sheets should be roads, railroad or trails. If this be impracticable, then well defined physical feature, as streams or cutting through brush and timber.

VERTICAL CONTROL.

145. Over the main traverse lines it is advisable in most cases to run a line of levels. (Par. 205.)

SUBSIDIARY CONTROL.

146. Each topographer now proceeds to complete the control of his area. His procedure will be as follows: Beginning at a convenient "fixed" point in his area, he runs, or causes to be run by his assistants, flying transit and stadia, compass and notebook, or sketching board traverses; using stadia or aneroid or clinometer for carrying elevations, as may be required by the length and importance of the line and as preferred by the individual. In this manner the topographer searches out the complete network of roads, trails, and streams for the entire field sheet on which he is engaged. This network should be so complete that the **contour sketcher** will need do no adjusting in his work. This will require that the distances between **subsidiary traverse** lines or other fixations should not exceed 1 mile on the scale of 1 mile to the inch or **in general 1 inch on the map whatever the scale.**

147. It is to be understood, of course, that the subsidiary traverses run by the topographers between points of higher control, etc., must not be plotted on the topographers record sheet until they have been properly adjusted. In long important lines, the adjust-

ment will be made by latitudes and departures, as described for main traverse lines. In the shorter lines, these computations are not justified and the graphical methods indicated in figures 12 and 13 will be used.

148. Figure XII shows method of graphical adjustment for a traverse run between two points, the positions of which are already plotted on the *record sheet.* Proceed as follows:

Plot the traverse, as actually measured in the field. In plane table work, of course, this is already done. Draw a line A B from the beginning point to the ending point of the plat. Now from the record sheet measure the straight line distance between the *record* positions of A and B. Measure off this distance from A on line A B. Call the end of the distance *b.*

Take any convenient point O and draw O A and O B. Now from *b* draw line parallel to O A. Where this line crosses O B is new position (B′) for B. From B′ draw line parapllel to A B. Where this line intersects A O is new position of A.

To find new position of station 1, draw line from A′ parallel to A1 and where it intersects O1 is now position (1′) of 1. From 1 draw line parallel to 1–2, etc.

The traverse A′, 1′, 2′–B′ is now adjusted and can be traced off onto record sheet.

149. Figure XIII shows *method when* a *circuit* is run from a known point back to the same point. Plat the traverse as measured in the field. The point A′ should be at A, but, due to errors, does not so plat. On a long straight line O A lay off from O, in succession, the lengths of the courses (A–1, 1–2, 2–3, etc., ——— 13–A′). From the end of this line lay off in any convenient direction the line *A B* equal to the error in closure A′A. Connect the outer end of this offset line to O. Now from each succeeding station point on the *long line,* draw a line parallel to the *offset line* A B.

Returning to the *plat* of the *traverse,* draw through each plotted station a line parallel to the final closure line A′A and in the direction of the closing station. On each line lay off its respective offset length, giving new positions for each station. Connect these new stations and the traverse is adjusted.

SKETCHING.

150. The field sheet which has been thus prepared consists of a network of traverses along roads, trails, streams, and across country with the horizontal detail along these traverses, with stations and easily identified objects shown in their proper adjusted locations and with their proper adjusted elevations given. It is the contour sketcher's task to go over the area assigned to him, sketching the drainage, culture, and forms of relief; generalizing the features to suit the scale and purpose of the map. The course in sketching will have taught the sketcher *how* to sketch rapidly. He must, on this work, follow similar methods, but "watch his step," as the map must answer severe tests. Especial care must be taken to see that all prominent landmarks are so located that they may be later used as orientation points in sketching or map reading.

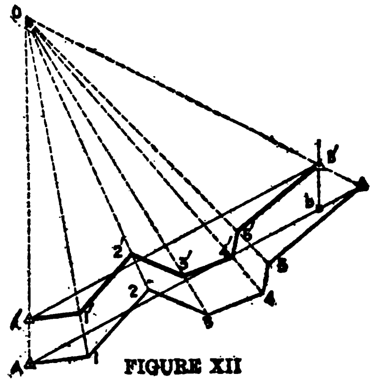


FIGURE XII

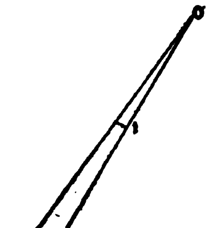
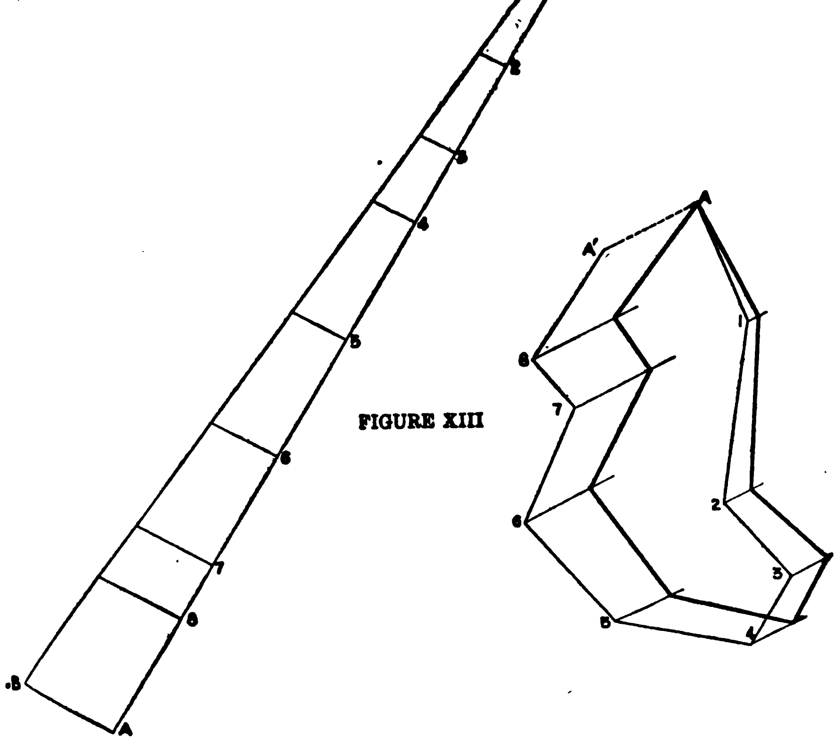



FIGURE XIII

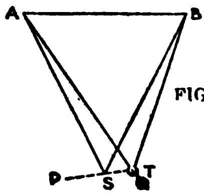


FIGURE XIV

LESSON XXII.

MONUMENTS, OFFICE RECORDS, AND DRAFTING.

151. Any properly made survey consists of three distinct parts. These are: (*a*) **Permanent marks or monuments on the ground,** (*b*) **complete records of the field work, properly filed,** and (*c*) the published sheets or copies issued for use.

(A) MONUMENTS.

152. A map represents to scale a certain portion of the earth's surface, everything shown being presumably properly located with respect to everything else, but if it ever becomes necessary to add new matter to the map or if we wish to extend it over a greater area or to incorporate it as part of a larger survey, we must be able to locate the **same points** both on the map and on the ground. Only a limited number of points in any map have their positions with relation to each other determined with great care. In the case of triangulation these are the triangulation stations; if the survey was controlled by traverses, they are the traverse stations. If, then, we put in permanent marks at each triangulation station, or, in the case of traverse control, permanent marks at certain traverse stations and **also show these stations on the map,** the relation between the map and the ground is established for as long a time as the monuments are undisturbed.

For the same reasons it is also essential to leave *permanent* level bench marks on the ground and to show their location on the map.

(B) OFFICE RECORDS.

The real map is the office record, the maps issued being copies of it. The office records consist of the field-instrument notes, the projection sheets on which the control is plotted, and the field topographical sheets. All office work should be plotted on double-mounted drawing paper, which is little affected by atmospheric changes and practically uniformly in all directions. These records must be filed so as to be easily accessible at all times. Double mounted paper, or, for well-controlled work, single mounted paper or vellum is used for field topographical sheets. Celluloid is very useful for field topographical sheets in regions where there is much rain or dew.

(C) PUBLISHED COPIES.

Copies issued for use may be blue prints from a tracing, black prints from a paper negative, or lithographic copies. For any large issue the usual military practice will be to print from zinc plates. (*See* Map reproduction.)

DRAWING.

153. The essential requirements of a good topographical drawing are *accuracy* and *clearness*. By accuracy is meant a faithful exhibit of measurements and observations made in the field, or of data taken from other maps. Clearness involves absence of confusion or crowding, and neatness in execution. *Beauty* and *pictorial effect* are ob-

tainable by skilled draftsmen only, and while always desirable, are rarely necessary. Persons who are not skilled draftsmen should not attempt pictorial effect, as it will detract from accuracy and clearness without substituting anything of equal value.

Avoid unnecessary haste in plotting and drawing. If possible, take time to check carefully all azimuths and distances plotted and be sure they are exact. There should be no approximation on the drawing board. Although an observer may have simply guessed a distance to be 550 yards in the absence of other information, the plotter should be careful to lay it down at exactly 550 yards.

Start with clean paper and keep it as clean as possible. In the office, wipe off the instruments before using, especially rulers, scales, and triangles. Dust the drawing carefully before beginning work. Dust again when stopping and cover with a cloth or paper. If necessary, dust the drawing and wash the hands occasionally while at work.

Make all ink lines firm and very black.—A drawing to be made in ink is usually drawn first in pencil, and in such cases a very hard pencil (4H or 6H) is best. If the pencil drawing is to be traced, a softer and blacker pencil should be used, but must be kept well pointed.

India ink in stick form gives the best results, but the time required for proper grinding precludes its extensive use in military field work. The prepared india inks in liquid form are ready for use and are satisfactory. They must be kept well corked when not actually filling a pen. If the ink gets thick in the bottle so that it will not run freely from a fresh-filled pen, add a little water.

Papers.—Manila paper of cream or buff tint, usually called *detail paper*, is suitable for sketches and drawings which are to be traced or used in the field. Only the better grade stands erasing, and that imperfectly. This paper comes in rolls 36, 42, and 54 inches wide. It may be ordered by the pound or yard.

White drawing paper may be had in rolls or sheets mounted on muslin or unmounted. Whatman's cold-pressed fine-grain is most generally useful. It comes in sheets of names and sizes as follows: Royal, 19 by 24 inches; Imperial, 22 by 30 inches; Double Elephant, 27 by 40 inches; Antiquarian, 31 by 53 inches. Roll papers are 27 to 63 inches wide.

Sheet papers unmounted and kept flat are best for field topographical use.

If a blot drops on the drawing, take a piece of blotting paper, tear a corner or edge to expose a fresh surface, and hold it in the blot without touching the drawing until the surplus ink is absorbed. Then press a dry blotter firmly on the spot and let it dry thoroughly before attempting to erase. A piece of newspaper may be used instead of blotting paper, but should be slightly moistened to hasten the absorption. For a large blot several pieces may be required.

Erasers for ink are of steel or rubber. A steel eraser or penknife must be very sharp to give good results. An eraser of gritty rubber is most generally used. It is best to use an erasing shield of thin metal or celluloid, which exposes the area to be erased through one of the openings and protects the rest.

Tracing linen is usually *dull back,* having one side glazed and the other dull. Erasing can be done on the glazed side only. The glazed side is used for ink and the dull side for pencil work. The glazed side requires preparation before use to remove excess of paraffin, which prevents ink from running well and clogs the pen. Rubbing hard with fresh blotting paper is the simplest method.

Tracing paper is alike on both sides. It will not erase. Most varieties are less transparent than tracing cloth.

In tracing, it is helpful to use a dull-pointed instrument in the left hand—a stylus or top of a penholder—to press the linen against the drawing at the point where the pen is resting.

ENLARGEMENT AND REDUCTION.

154. The simplest method is by squares. Divide the original into squares of 2 inches or less by lines drawn parallel to the borders. Divide the paper on which the copy is to be made into squares with sides corresponding to the same distance on the scale of the copy that the side of a square on the original itself does to the scale of the original. If a plotting scale of the original be placed on the side of a square on the original and the plotting scale of the copy on the side of a square of the copy the readings should be the same. The square on the copy will be larger if the drawing is to be enlarged and smaller if it is to be reduced. The ratio between the sides of the squares on the original and the copy is the ratio of reduction or enlargement. This ratio must not be confused with the ratio of areas of the two maps, which is different and not important.

Select a square of the original and reproduce its contents in the corresponding square of the copy, or take a feature of the original, as a road or stream, and trace its course through several squares.

Usually the position of a point in a square or on one of the sides can be *estimated* with sufficient accuracy. Important points may be located by measurement of distances from the nearest sides of the squares, using the scale of the map and the scale of the copy, respectively.

Instead of drawing the squares on the original, they may be drawn on tracing linen or paper laid over it, or fine threads may be stretched to form the squares. Every drawing board should have a scale of inches on each edge marked with fine saw-cuts or with small tacks to facilitate the drawing of squares.

CHARACTER OF WORK.

155. All lines must be clear, sharp, and distinct, drawn or printed in jet black, waterproof ink. Colors will not be used except when clearness of representation absolutely demands their use; special pains will be taken to avoid them in drawings intended for publication. So far as practicable, uniformity of size must be maintained in figures or letters presenting a particular kind of information, such as soundings, elevations. names of proprietors, names of minor towns, names of counties, etc., and figures and letters must be clear, distinct, and readily legible, especially on drawings intended for publication, in which case the letters and figures must be made of size large enough to avoid blurring and obscurity when reduced by

photolithography. The use of type for letters and figures and of films or rollers for representing typographic features is recommended.

The title should be placed in the lower right-hand corner, unless use of this space for other matter is absolutely unavoidable. If practicable, the size should not exceed 5 inches by 5 inches. The use of such words as "Map of," "Plan of" is thought to be unnecessary. The approval of the officer in charge will be placed near, and preferably at the side of the title. At the upper left-hand corner (and outside of the border, if a border is used) the words "War Department" will be placed; and at the upper right-hand corner, in similar position, the wording "Corps of Engineers, U. S. Army," will be placed. When the title is not placed as above, a brief title will be placed outside the border, on lower right-hand corner, to identify the drawing.

A graphical scale should be shown in all cases, and a statement of the scale may be added when useful. Drawings submitted for publication must be prepared with a view to reduction to the smallest practicable compass, and they must bear a statement in pencil, on the lower margin, of the amount of reduction contemplated. All maps are usually reduced at least one-third.

The true meridian should be shown on all maps or charts of land or water areas, if known, and the magnetic declination should also be given.

In the case of topographic or hydrographic surveys, a brief description should be placed upon the map or chart, in tabular form, of the principle triangulation points or bench marks, including the general location and character of the monuments or bench marks, the coordinates and the elevation. Whenever possible the direction of water flow of all waterways must be indicated by arrowheads pointing in the direction toward which the water moves. Tidal flow must be indicated by double or multiple arrowheads.

The following are the standard topographical symbols and styles of lettering to be used on all maps:

ABBREVIATIONS.

A........Arroyo.	G. S.....General store.	Pt......Point.
abut....Abutment.	gir......Girder.	q.p......Queen-post.
Ar.......Arch.	G. M....Gristmill.	R.......River.
b.........Brick.	I........Iron.	R. H...Roundhouse.
B. S.....Blacksmith shop.	I........Island.	R. R....Railroad.
bot.......Bottom.	Jc.......Junction.	S.......South.
Br.......Branch.	k.p......King-post.	s.......Steel.
br........Bridge.	L........Lake.	S. H...Schoolhouse.
C........Cape.	Lat......Latitude.	S. M...Sawmill.
cem......Cemetery.	Ldg.....Landing.	Sta.....Station.
con.....Concrete.	L. S. S..Life-saving station.	st......Stone.
cov.....Covered.	L. H...Lighthouse.	str.......Stream.
Cr.......Creek.	Long....Longitude.	T. G....Tollgate.
d.......Deep.	Mt.....Mountain.	Tres....Trestle.
cul......Culvert.	Mts......Mountains.	tr........Truss.
D. S....Drug Store.	N.......North.	W. T...Water tank.
E........East.	n. f.....Not fordable.	W. W..Water Works.
Est......Estuary.	P.......Pier.	W......West.
f........fordable.	pk......Plank.	w........Wood.
Ft........Fort.	P. O...Post office.	wd.....Wide.

Bridge

Indicate character and span by abbreviations.

Example: w.k.p. 40 x 20 / 10

Meaning wooden kingpost bridge, 40 feet long, 20 feet wide, and 10 feet above the water.

Streams

Indicate character by abbreviations.

Example: 15'wd. x 8'd. n.f.

Meaning a stream 15 feet wide, 8 feet deep, and not fordable.

House Church School house S.H.

Woods Woods Orchards Cultivated Land Cult.

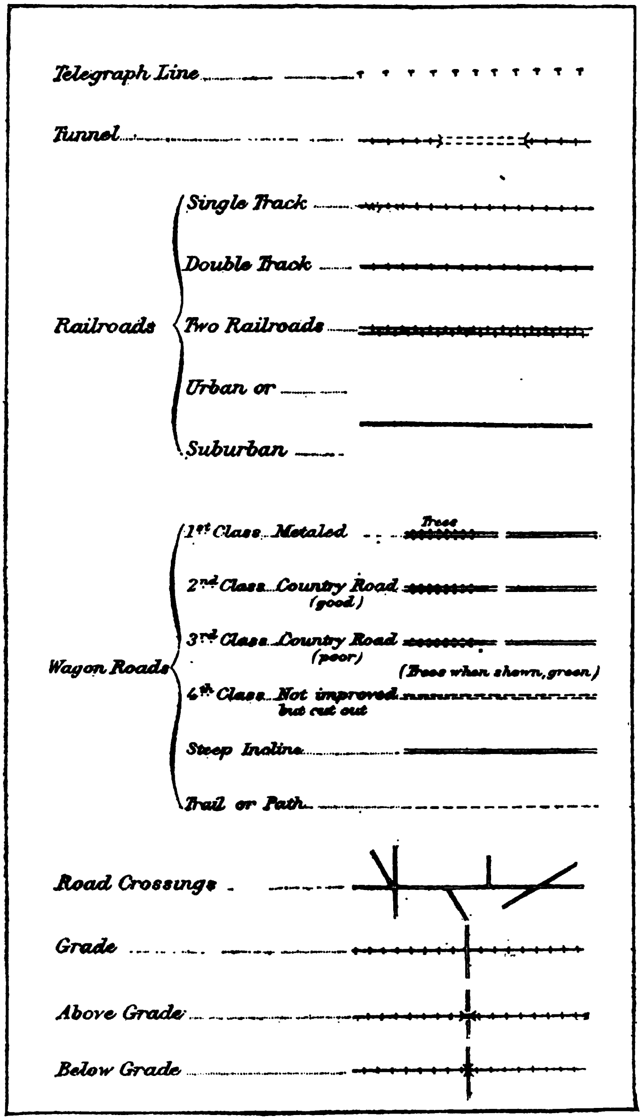
If boundary lines are fences they are indicated as such.

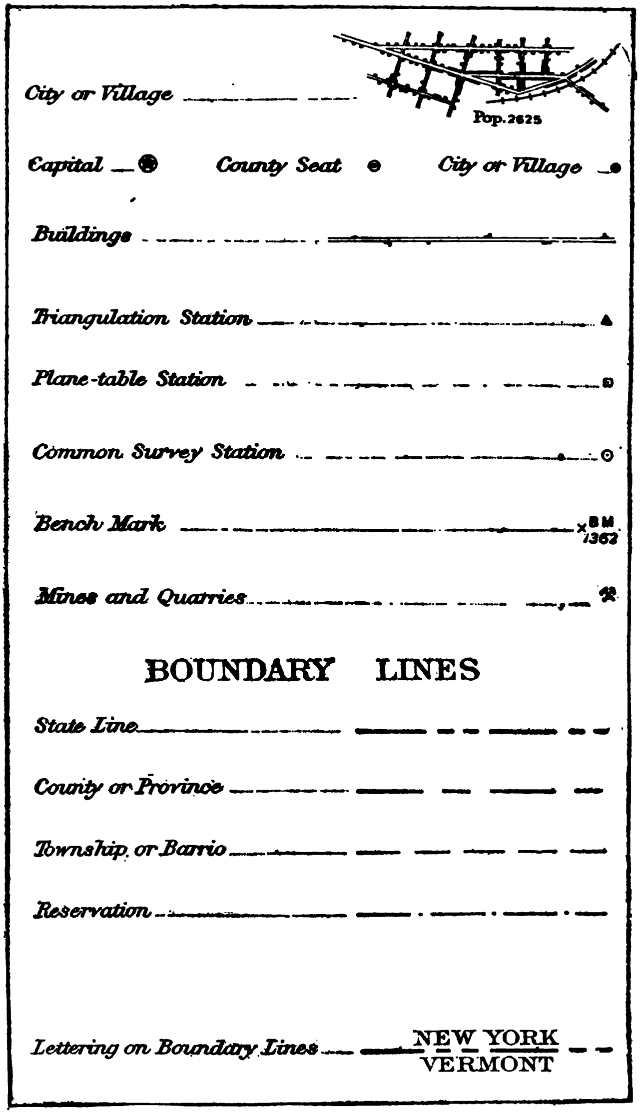
Brush, crops or grass, important as cover or forage Brush, corn, grass, etc.

Cemetery Trees, isolated

Cut and fill — Cut 10' cut 10 feet deep

Fill 10' fill 10 feet high


Telegraph Line
Tunnel
Railroads
Single Track
Double Track
Two Railroads
Urban or
Suburban
Wagon Roads
1st Class Metaled
Trees
2nd Class Country Road
(good)
3rd Class Country Road
(poor)
(Trees when shown, green)
4th Class Not improved
but cut out
Steep Incline
Trail or Path
Road Crossings
Grade
Above Grade
Below Grade


City or Village
Pop. 2625
Capital
County Seat
City or Village
Buildings
Triangulation Station
Plane-table Station
Common Survey Station
Bench Mark
BM 1362
Mines and Quarries
BOUNDARY LINES
State Line
County or Province
Township or Barrio
Reservation
Lettering on Boundary Lines
NEW YORK
VERMONT

Medical Corps

Ordnance

Signal Corps

Engineer Corps

Gun Battery

Mortar Battery

Fort } *True plan to be shown if known*

Redoubt

Camp

Battle

Trench

OBSTACLES

NOTE: When color is used execute these in red

Abattis

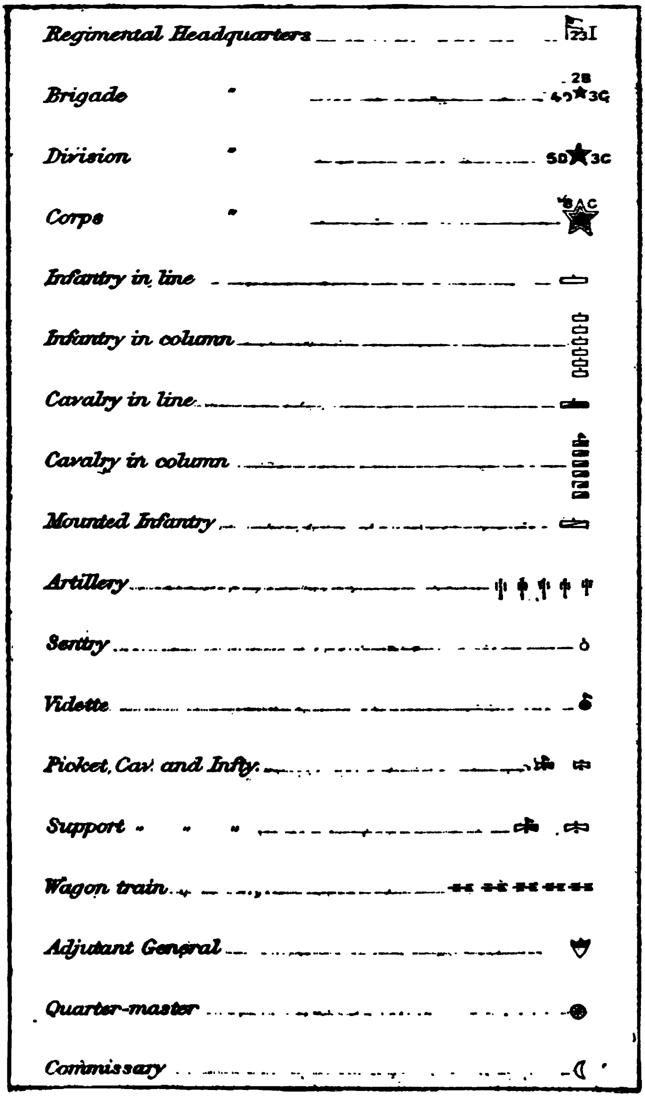
Wire entanglement

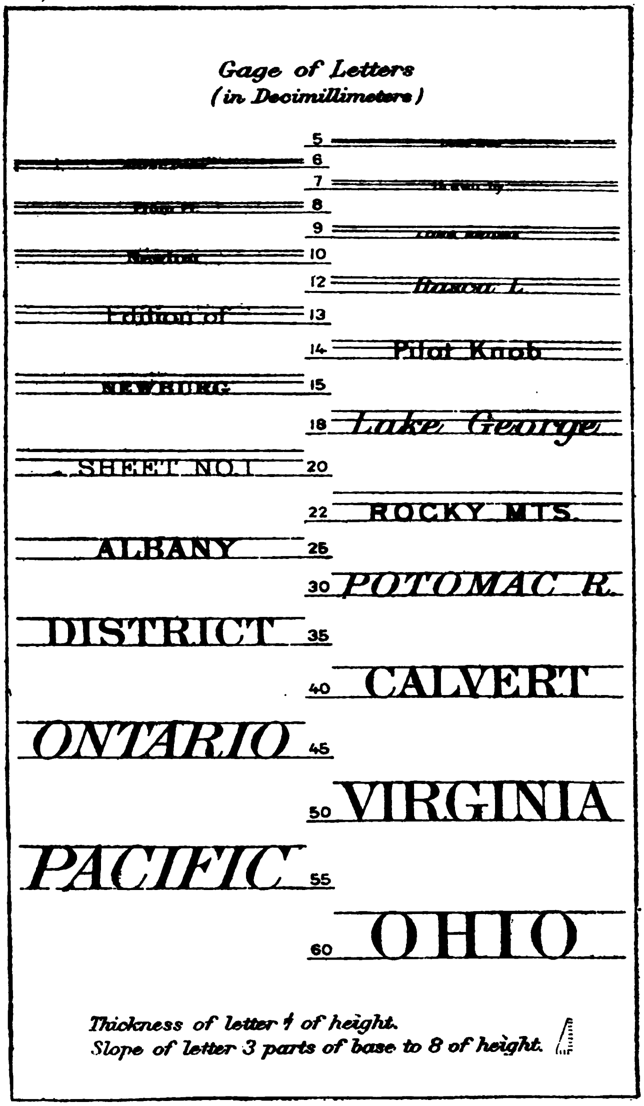
Palisades

Contact Mines

Controlled Mines

Demolitions


Regimental Headquarters
Brigade "
Division "
Corps "
Infantry in line
Infantry in column
Cavalry in line
Cavalry in column
Mounted Infantry
Artillery
Sentry
Vidette
Picket, Cav. and Infty.
Support " " "
Wagon train
Adjutant General
Quarter-master
Commissary


Gage of Letters
(in Decimillimeters)
5
6
7
8
9
10
12
13
Edition of
14
Pilot Knob
15
NEWBURG
18
Lake George
20
SHEET NO. I
22
ROCKY MTS.
25
ALBANY
30
POTOMAC R.
35
DISTRICT
40
CALVERT
45
ONTARIO
50
VIRGINIA
55
PACIFIC
60
OHIO
Thickness of letter ⅐ of height.
Slope of letter 3 parts of base to 8 of height.

HYPSOGRAPHY

Mountains, Plateaus, Lines of Cliffs and Canyons (all capital letters)

ABCDEFGHIJKLMNOPQRSTU VWXYZ

Peaks, small Valleys, Islands and Points (with Cap. initials)

abcdefghijklmnopqrstuvwxyz

PUBLIC WORKS

Railroads, Tunnels, Bridges, Ferries, Wagon-roads, Trails, Fords and Dams (capitals only)

ABCDEFGHIJKLMNOPQRSTUVWXYZ

CONTOUR NUMBERS

Heavy contours 1234567890

Light contours 1234567890

MARGINAL LETTERING

ABCDEFGHIJKLMNOPQRSTU VWXYZ

(with Cap. initials)

abcdefghijklmnopqrstuvwxyz

1234567890

1° R. F. $= \frac{1}{100} = 8'.33$ to $1'' = 633''.6$ to 1 mile.

2° R. F. $= \frac{1}{120} = 10'$ to $1'' = 528''$ to 1 mile.

3° R. F. $= \frac{1}{500} = 41'.66$ to $1'' = 126''.7$ to 1 mile.

4° R. F. $= \frac{1}{600} = 50'$ to $1'' = 105''.6$ to 1 mile.

5° R. F. $= \frac{1}{4224} = 352'$ to $1'' = 15''$ to 1 mile.

6° R. F. $= \frac{1}{5280} = 440'$ to $1'' = 12''$ to 1 mile.

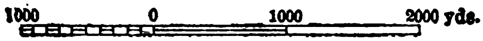
7° R. F. $= \frac{1}{10000} = 833'.3$ to $1'' = 6''.34$ to 1 mile.

8° R. F. $= \frac{1}{10560} = 880'$ to $1'' = 6''$ to 1 mile.

9° R. F. $= \frac{1}{20000} = 1666'.7$ to $1'' = 3''.17$ to 1 mile.

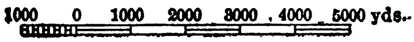
10° R. F. $= \frac{1}{21120} = 1760'$ to $1'' = 3''.00$ to 1 mile.

11° R. F. $= \frac{1}{52800} = 4400'$ to $1'' = 1''.2$ to 1 mile.

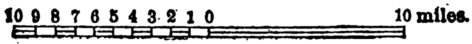


12° R. F. $= \frac{1}{63360} = 5280'$ to $1'' = 1''.00$ to 1 mile.

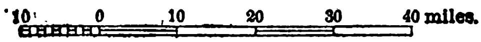
13° R. F. $= \frac{1}{126720} = 10560'$ to $1'' = 0''.50$ to 1 mile.

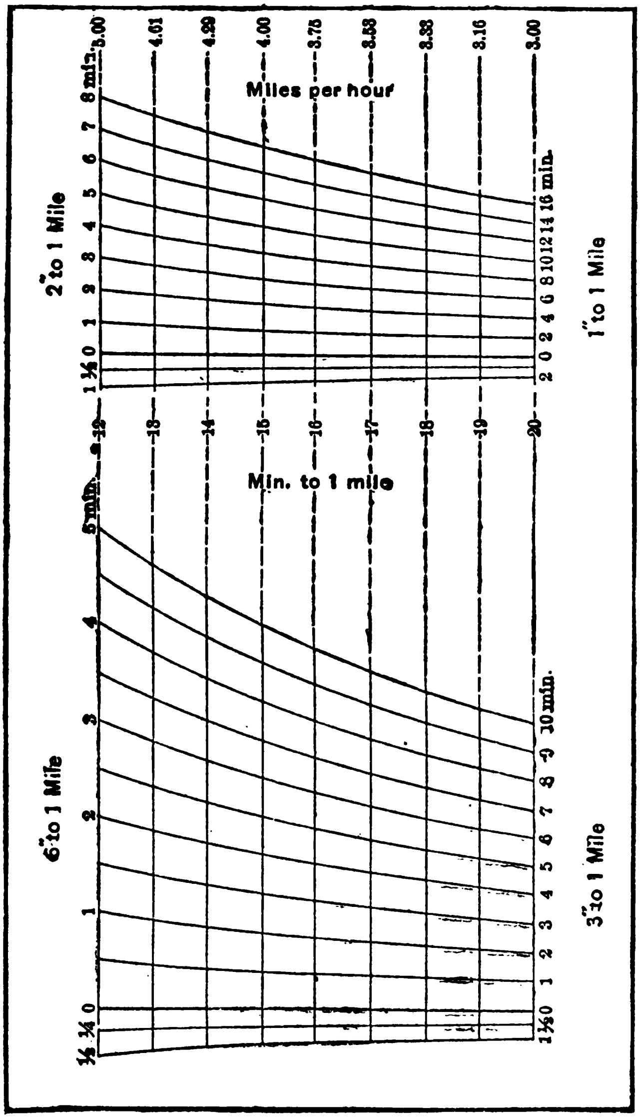


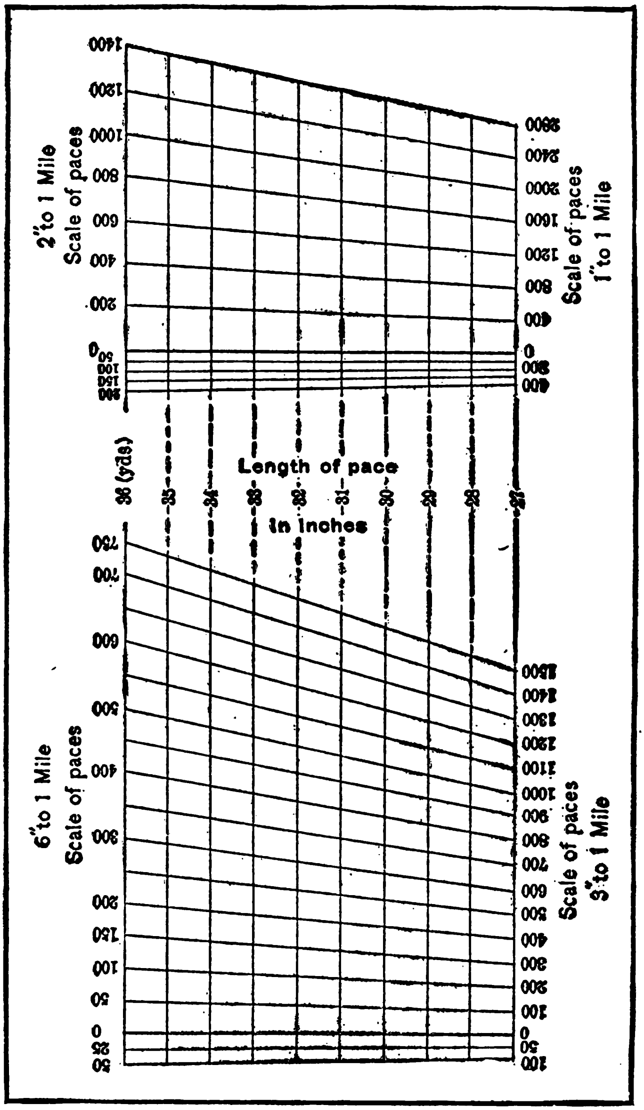
14° R. F. $= \frac{1}{633600} = 52800'$ to $1'' = 10$ miles to $1''$.



15° R. F. $= \frac{1}{1584000} = 132000'$ to $1'' = 25$ miles to $1''$.




Miles per hour
Min. to 1 mile
2" to 1 Mile
1" to 1 Mile
6" to 1 Mile
3" to 1 Mile


2" to 1 Mile
Scale of paces
1" to 1 Mile
Scale of paces
Length of pace
in Inches
36 (yds)
35
34
33
32
31
30
29
28
27
6" to 1 Mile
Scale of paces
3" to 1 Mile
Scale of paces

LESSON XXIII.

MAP REPRODUCTION.

156. ***The hektograph*** is one of the most satisfactory means of reproducing maps, drawings, etc., in the field on account of the simplicity of the equipment. Staple compound for hektographs is sold by Frederick Post Co., Chicago, at 25 cents per pound and the hektograph can be readily made therefrom, or it can be purchased set up from the company. The only apparatus needed is the hektograph and hektograph ink and the paper necessary for impressions. The drawing must be traced with hektograph ink on a paper with a non-absorbent surface. The drawing is then placed ink side down upon the hektograph and gently rubbed until the ink has thoroughly taken on the hektograph material. This may be determined by lifting one corner of the drawing from time to time. When the transference is completed, impressions are made by laying a piece of paper on the hektograph and rubbing over it when it readily takes the impression. From 30 to 75 readable copies can be thus obtained. When the impressions begin to fail, the original drawing may be again applied to the hektograph, which has been washed off, and a number of additional copies secured. When the work is completed the surface of the hektograph is washed off with light touch of a damp sponge, all ink readily clears, and the hektograph is ready for further work or to be stored until needed.

The compound above referred to gives most excellent line impressions and is guaranteed to stand any climatic conditions. The inks come in a number of colors and added value attaches to a field process that will give you in one process reproduction in several colors.

Twenty-five readable impressions have been obtained from a sketch made with an ordinary purple copying, or indelible pencil.

The hektograph was considerably used by the Japanese for map reproduction work during the Manchurian War.

Photographic, brown print, black print, or blue print papers being available, reproduction of maps or drawings from photographic plates, films, or tracings can be readily made. Using the thinner brown or black print paper, negatives can be prepared, and the speed of production of prints increased by printing from these and they also furnish a most excellent form in which to store such work for future reproduction. With most of the above papers it is possible to print direct from an original drawing by greasing it and rendering it thereby translucent. A most excellent grease for this purpose is so-called "banana oil."

The brown and black print papers are developed by washing only, and are *fixed* by a weak hypo solution. Blue print paper is developed by washing and no fixing is required. The photographic papers will be handled exactly as in ordinary photographic work, employing the same developer and fixing bath.

157. **Zincographic processes.**—The method of reproduction of most general application is the zincographic process, which may be employed in two ways—the photographic and autographic.

158. **The autographic process** has the advantage of being entirely independent of light. It consists in drawing on transfer paper (a

trade article) with autographic ink (a trade article which can be obtained in either liquid or stick form) and transferring this drawing to the surface of the zinc. This transfer is accomplished by placing the transfer paper, after completion of the drawing, between dampened sheets of blotting paper until it is thoroughly moistened, but not wet. The transfer paper is then placed, ink side down, upon the zinc plate, the latter having been given a slight grain by immersion for one minute in a solution of 1 gallon of water to 2 ounces of commercial nitric acid and 1 ounce of alum, after which it is dried. The zinc plate with the transfer paper upon it is then run through the lithographic press with a very light pressure, just enough to straighten the paper out upon the plate. The pressure is then gradually increased until a maximum is reached at the fifth run through. Take the plate out and run it through the press in the opposite direction five times, increasing pressure as before. Moisten the transfer paper with a damp sponge and remove it from the plate, when the ink will be found to have entirely left the paper and the image will appear on the surface of the plate. Wash the plate to remove the glue particles which were transferred from the glossy side of the paper (upon which side the drawing must be made, care being taken not to touch it with the hands) and which will adhere to the zinc and which, if left on the plate, will cause trouble later by taking up the etching powder. The plate will stand hard scrubbing with a sponge, after which it should be wiped off with a damp chamois skin and then dried by natural or artificial heat.

The lines now should be built up a little and this accomplished by dusting the plate, which should be at about the same temperature as its surroundings, with etching powder. This powder will adhere to the ink lines and should be brushed from the clean surface of the zinc with a camel's hair brush or a tuft of dry cotton. An overheated plate will take etching powder everywhere. The lines of the plate now appear red and the plate should be heated (gas or kerosene stove) until the color changes to a brilliant black, indicating combination of the ink and powder. In heating the plate will warp slightly, and it should be placed on a flat surface to cool. Now look over the plate and remove with a fine needle any spots of etching powder which may have been burned in on the whites of the plate.

The plate is now ready for the first etching, which means the immersion of it in a tray containing a solution of 5 ounces of commercial nitric acid, 38 per cent, in two gallons of water. It should remain in this solution for 1½ to 2 minutes, the tray being continually rocked to keep the acid in motion across the plate, and the surface of the plate being brushed lightly with a bristle brush to insure a fresh surface upon which the acid may act. At the expiration of the 1½ to 2 minutes, the plate is removed from the etching bath, immediately washed thoroughly, if possible under running water, the surface being gone over with a wad of absorbent cotton. It is rare that one etching will give a satisfactory plate, and it will usually be found desirable to now dry the plate, again powder with etching powder, heat in and etch for from 1½ to 3 minutes. Upon again being thoroughly washed the plate, while still wet, is flowed with the dextrine solution, and then placed on an inclined surface to drain and dry. This takes one to the process of printing from the

plate, and as this is the same for all plates from this stage on, we will now pass to the photo-zincographic process.

159. **Photo-zincographic process.**—This process is the sensitizing of the zinc plate and the transfer thereto of impressions by the action of light through some medium immediately superposed upon the zinc plate. The zinc used for this purpose is No. 19 gauge and is usually purchased highly polished. If the plate is not polished this must be done, and in any event it is always necessary to go over the polished plate with a willow charcoal polisher, which should be kept under water when not in use. Go over the plate with an even stroke, always keeping the same direction and not applying too much pressure. The same object may be accomplished by using powdered pumice, applying with a tuft of cotton. The plate must be then thoroughly washed and is ready to receive the sensitizing solution. If instead of a new plate, a plate that has already been printed from is being used, first remove all ink therefrom by means of turpentine, go over the plate with lye to remove all grease, and scrub down the slightly relieved lines of the former image with a scotch hone. The plate is then polished, as before, with charcoal or pumice.

A satisfactory medium speed sensitizing solution, which can be used in any climate, is the following:

The white of one fresh egg, or 120 gr. dry albumen; bichromate ammonia (C. P. Bichromate), 15 gr.; water, 7 oz.

Beat the egg with an egg beater, or dissolve the dried albumen in the 7 ounces of water; pulverize the dichromate with mortar and pestle and mix it with the water; add a few drops of 28 per cent ammonia water until the solution assumes a clear yellow color. This solution is not affected by light as long as it is in liquid form, but is sensitive to light when dry.

Rinse the now polished zinc plate under the tap, or when running water is not at hand squeeze a wet sponge over it and allow the water to run off the plate. Then, while the plate is still wet pour the sensitizing solution over the plate by gliding slowly with the glass containing the solution along the upper edge of the plate which is held on the left hand in an inclined position. Allow the surplus solution to run off the plate and then repeat the operation having one of the formerly inclined edges of the plate uppermost. A third pouring of the solution over the plate is sometimes recommended. Watch out for dust particles and air bubbles during the sensitizing of the plate.

The plate is now placed in the whirler and whirled over a heater to give an even distribution of the solution and also to dry the plate. Do not whirl too fast at first as this may throw too much solution from the edges of the plate and make the corners come weak. While whirling, keep the plate also swinging in order as far as possible to give even heating over the entire surface. As soon as the plate begins to dry a dark circle will appear in the center and will almost immediately extend to the edges if the plate has been properly handled. The plate is now removed from the whirler and, being sensitive to light, is placed in a dark place to cool.

The plate, having cooled, is put into the printing frame, sensitive side toward the glass, with the maduro negative, stripped film, or tracing between it and the glass, and so placed as to give the reversed

impression on the zinc plate. Absolute contact between plate and negative is most important and consequently the printing frame and glass are heavy and considerable pressure is applied. While the sensitized plate is affected by light, it is entirely practicable to carry out the above-mentioned operations without hurry in the light of an ordinary room, without damage to the plate. It is only really sensitive to the rays of sunlight or intense artificial light.

The plate is now exposed to sunlight or artificial light and by this action the albumen reached by the light rays is rendered insoluble in water, while that not so acted upon remains soluble. The time of exposure is dependent upon so many conditions that it must be arrived at by a beginner by experiment. As a guide it may be stated that a zinc plate sensitized as above and exposed under a perfect negative to the rays of the sun during July and August between 10 in the morning and 3 in the afternoon in the central part of the United States will take a 1½-minute exposure if the drawing to be produced consists of ordinary heavy lines.

After exposure the plate is taken from the frame and laid on a level surface and immediately rolled up with etching ink. The surface upon which the plate is laid must be absolutely a plane or distortion will result. An ordinary lithograph stone makes a most excellent bed for this purpose, but a heavy block of wood with care may be made to serve. The exposed plate is evenly coated with the etching ink, rolling it up in two directions at right angles to each other. Very little ink is used and the yellow of the film should always show through the ink.

Etching ink is a trade preparation resembling ordinary black lithographic ink, but it has a greater proportion of resinous material and less coloring matter. A very small quantity, about as much as can be placed on a 5-cent piece, is worked up with the mixing knife and then spread upon the ink slab (which must be plane surface) and rolled out with the roller until both slab and roller show an even coating of ink. If the ink is too hard to work up it may be thinned with a drop of aniseed oil, but great care must be used to thin the ink only enough to make it work, otherwise it will not have sufficient body when applied to the plate. Rollers must be frequently washed with turpentine to keep in proper condition.

The preparations for applying etching ink are supposed to have been made prior to exposing the plate, as any delay in working up the plate should be avoided.

We now have a zinc plate covered with a bichromatized albumen film, portions of which, corresponding to the lines, etc., on the original drawing, have been rendered insoluble in water by the action of light. The whole surface has then been given a coating of ink. The plate is now washed, rubbing the surface gently with a tuft of cotton. If the plate has been correctly exposed, the albumen corresponding to the whites of the original drawing will be readily removed, taking with it the ink, and there will be left the reverse of the drawing in ink lines on an otherwise clean zinc plate. If the plate has been underexposed the lines will begin to disappear early in the washing and another plate with longer exposure should at once be tried. If overexposed the ink will smear over the plate and the whites will not come up. A slight overexposure may sometimes

be corrected by adding a few drops of stronger ammonia to the water in the washing tray.

As the zinc plate will usually be larger than the negative used in exposure the outer edges will have been exposed and rendered insoluble and will show a black frame around the reproduction. This must be removed by scrubbing with pumice stone and a cotton tuft.

Take the plate from the tray and dry it by patting it with a damp chamois skin; do not wipe it as the lines may smear. Look carefully over the reproduction, and if there are any lines missing paint them in by means of a little etching ink moistened with turpentine and applied with a red sable brush. Also remove any unnecessary ink spots on the plate. Be careful in this work not to touch the plate with the arm or hand for fear of smearing.

From this point on the development of the plate is the same as in the autographic process and will not be again described.

PRINTING FROM ZINC PLATES.

160. While the plates have been etched so as to give the image a slight relief the main principle involved in the printing from zinc plates is that "grease attracts grease and is repelled by water."

The gummed plate, having dried, is wiped over with a damp rag to remove surplus dry gum. After that it is gone over before each rolling up with a rag moistened in water with a little dextrin in it to keep the plate from drying too rapidly. The proper proportion is usually obtained by taking about one part of the solution for gumming plates and mixing it with eight parts of water.

A small portion of lithographic ink is thoroughly worked up with an ink knife. (This ink slab may be a piece of polished marble or a lithographic stone.) The ink is then thoroughly distributed by the roller. Never put ink directly upon the roller. Do not put varnish in the ink, but be careful to have the roller saturated with varnish.

The roller being now inked up, the plate, which must be on a perfect plane surface, is wiped over with the damp rag (dextrin mixture), and the roller is passed once over and back, quickly if the ink is soft and liquid, slowly if the ink is very stiff. The gummed portion of the plate takes moisture from the rag and will reject the greasy ink upon the roller. The lines being greasy reject the wetting and take the ink. When first working up a plate it should be inked and wiped several times, and it may then be necessary to make a half dozen impressions before a satisfactory one is secured. If an ordinary sliding contact hand press is used, the plate can be inked on whatever is used as the bed of this press. If the clothes-wringer press is used, a separate surface will have to be provided for inking up the plate. To get an impression the paper is laid upon the plate immediately after rolling up, it is passed through the press, and when taken off it will show the impression.

Avoid flat and glossy paper for printing purposes. If you have to use it fan the plate dry after each rolling up or the paper will stick.

If in rolling up some of the whites take ink, it is because, due to carelessness, the entire plate was not covered in the wiping with the damp rag. Go over it again with the damp rag and then give it a very quick roll and the ink will be removed. If the plate blocks up

and the lines of the impression become blurred, gum the plate with a rather thick solution of dextrin, and, while the dextrin is still liquid, wash off the plate with turpentine, being careful that no pressure is exerted upon the lines. The ink being removed keep the plate for several minutes under running water in order to remove the turpentine. Then gum up with the ordinary solution of dextrin and let the gum dry. The plate prepared in this way can be kept for years.

There is almost no limit to the number of impressions that can be obtained from the good plate which is taken proper care of. In the choice of papers to be used, some discretion is necessary if very particular results are to be obtained, and the papers should be selected after experiment has established their suitability.

Direct methods.—It may not always be practicable to make a brown print negative of the drawing to be reproduced, owing to lack of paper, nor may it be possible to make and strip a photographic negative. The reproduction can be made directly from a tracing, or if the drawing be on opaque paper this may be rendered translucent by the application of "banana oil," or a mixture of 3 parts castor oil and 10 parts alcohol. The impression on the zinc plate may be obtained by one of the following methods:

(*a*) 1. Prepare sensitizing solution:

Albumen	1 oz.
Water	6 oz.
Ammonium bichromate	27 grams.
Ammonia, 28 per cent	6 drops.

2. Sensitize a polished plate by "flowing" twice with the solution.
3. Dry by whirling over heat, being careful not to heat plate any more than is absolutely necessary.
4. Let plate cool.
5. When printing from tracing, give 40 seconds at 8.30 a. m., with good sunlight, decreasing time as sun gets stronger so that by 10.30 a. m. time will be from 25 to 30 seconds.
6. When printed, roll up with stiff etching ink. Put on thin coat of ink and smooth out with smooth leather roller. Ink and roller should be free from dust, or pin holes will result.
7. Place plate under water tap until all lines are clear. Going over surface with a tuft of cotton while holding under tap will assist in removing ink from lines.
8. Immerse four seconds in solution of 2 ounces nitric acid to 3 gallons water.
9. Remove from solution and wash thoroughly to remove acid.
10. Dry plate by patting with chamois skin. Dry back of plate with rag.
11. Warm slightly over stove to complete drying of lines.
12. Roll up with the same etching ink, using "composition" roller. Put on medium coat of ink and roll into lines with smooth leather roller, or by continued rolling with composition roller without the addition of any more ink.
13. When lines show black through coating of ink place plate in a 10 per cent solution of 28 per cent acetic acid and water. Rock tray for one or two minutes, then start removing ink from ground of plate by dragging medium-sized tuft of cotton over surface. Plate should be clear in five to seven minutes.
14. When clean remove and rinse, then dry by whirling over heat, let cool, then powder up, burn in, etch, and print as usual.

(*b*) 1. Expose a sensitized zinc plate under the drawing.
2. Develop in the usual manner. This gives white lines on a black ground.
3. Etch the plate long enough in the nitric acid bath to give a slight depth to the lines.
4. Wash and dry.
5. Rub ordinary asphaltum paint into these etched lines and dry.

6. Scour whole surface of plate with charcoal stick. The lines which contain the asphaltum paint, being deeper than the remainder of the plate, are not affected by the charcoal stick. Thus there is obtained a polished plate bearing the impression in depressed lines filled with asphaltum paint.
7. Etch the plate again. This takes down the whites and leaves the asphaltum lines in relief.
8. Gum up and dry, ready for use.

Excellent results may be obtained by this method, which, however, takes a little longer than when using a negative, on account of the time required to scour the plate.

Field expedients.—It is entirely possible that work which will give satisfactory rough results in the field can be done with a much reduced apparatus. For instance, it has been found that impressions can be gotten without the press by simply laying the paper on the inked zinc plate and passing a rubber roller over it so as to press upon all parts. There is a tendency for the paper to shift and give a blurred impression, but this can be sufficiently avoided by a little care, thus disposing with the press.

It has also been found that a satisfactory exposure can be made by placing the negative upon the sensitive side of the zinc plate, fastening it in any convenient manner, then bowing the sensitive side of the plate to the front, which will stretch the negative and give fair contact. During exposure the plate must be so moved as to give all portions an even illumination. The above are, of course, makeshifts and will not give such results as are usually desired, but they may save the day in an emergency.

Negatives.—If the original be a tracing the best negative is a brown print. This negative is made in the usual manner on thin paper and is placed brown face down upon the zinc plate.

Photographic negatives: 1. Make the usual photographic negative on a "stripping" plate, strip off and place, reversed, upon a clear plate. To print on the zinc plate, place film side down, thus giving reversed impression. 2. Expose a "process" photographic dry plate in the camera backward—that is, with the emulsion side away from the lens, correcting the focus for the thickness of the plate. Develop as usual. To print on the zinc plate, place emulsion side down. 3. The most satisfactory method of producing photographic negatives for lithographic work is the "wet-plate" process. This, however, is not adapted to field purposes. 4. The cameragraph: This apparatus will make at a single operation a positive negative brown print by which to transfer any drawing to a zinc plate. It consists of a copying board on which to place the map or drawing to be reproduced, a camera fitted with a reversing prism, and special attachments for carrying a roll of brown print paper, a developing and a fixing bath. To produce a negative, sufficient paper therefor is fed down from the roll into the focal plane and the exposure made. By means of a mechanical attachment this exposed piece is cut off the roll and is then fed through the developing and fixing tanks, removed, and dried. Full directions for operation come with the cameragraph.

PART II.

DETERMINATION OF AZIMUTHS.

161. **The compass** is the standard instrument for the determination of **azimuths** in topographical reconnaissances. It consists of case, needle, dial, pivot, and stop.

The dial may be fixed to the case or it may be **movable, that is,** moving with the needle to which it is attached. The **stop** raises the needle from the **pivot** and clamps it against the glass cover. A good compass must have a needle sufficiently magnetized to settle accurately and a **pivot** which is true. If the needle becomes too weak, it may be remagnetized by rubbing gently from **pivot to point** on a permanent or electromagnet, each end of the needle to be rubbed on the pole of the magnet which attracts it. In returning the needle for another stroke, carry it a foot or more from the magnet. **The pivot** may be polished with Putz pomade or a similar substance on a soft stick. **If possible, turn in a defective compass and get one in its place.**

A needle loses part of its magnetism if kept for a long time out of the plane of the magnetic meridian. **In storing a compass,** therefore, care should be taken to see that the needle is in the magnetic meridian with the N. end of the needle pointing north.

A symmetrical needle tends to point downward toward the nearer magnetic pole of the earth. This displacement from the horizontal is called **dip,** and is measured in degree of arc. Immediately over the magnetic poles the needle stands vertically or has a **dip** of 90°. Near the Equator, where North and South Poles of the earth exert an equal influence, the needle will be horizontal, or the **dip 0.**

For reading azimuths the needle must be kept in a horizontal plane, which is done by a small movable counterweight (to overcome the dip). For considerable changes in latitude, as in passing from the United States to the Philippines, the counterweight will require adjustment to keep the needle horizontal, and in passing from the Northern to the Southern Hemisphere, the counterweight must be changed to the other end of the needle.

There are two adopted forms of compass for topographical reconnaissance, one in which the **dial is fixed** to the case and one in which the **dial moves with the needle** to which it is fixed.

THE BOX COMPASS.

162. The **dial** or face on which the graduations are marked is rigidly attached to the case. The type of **box compass best adapted to running courses by azimuth** is constructed as follows: The graduations read counterclockwise continuously from 0 to 360; the instrument reads 0 when pointing south and 180° when pointing north; the E. and W. points, if marked, are reversed.

To determine the azimuth of a line point the north and south line of the case along the line (the north point away from the observer) and read the N. end of the needle. The dial is graduated to single degrees, but when the needle is stationary the reading can be estimated to half degrees.

Many box compasses are not graduated in the manner above dscribed. To use such compasses for azimuth reading they should

be altered to conform to the conditions cited. This is ordinarily done by pasting paper over the stamped numbers on the dial and renumbering in ink or pencil.

THE PRISMATIC COMPASS.

163. The dial containing the graduations is attached to the needle and moves with it. It is read by means of a small prism, adjustable for focus. This prism is mounted on a hinge joint and can be turned down for carrying. The line of sight of the instrument is determined by front and rear sights, which fold down when not in use, at the same time stopping the needle. The needle may be compensated for dip by a bit of sealing wax on the under side of the dial card. The graduations on the dial should be numbered so as to read azimuths, as above described, beginning at the south point. If the graduations are not so numbered, they should be altered as follows: The zero should be at the north end of the needle (which is on the under side of the dial) and the graduations should run clockwise continuously to 360°. **It is to be noted that with such numbering the instrument will not read azimuths if used as a box compass.** The index is a point on the case, and as the dial is movable the graduations are numbered clockwise, instead of counterclockwise, as in the box compass. **Readings should be made through the prism.**

To determine the **azimuth** of a line with this instrument, adjust the prism until the graduations on the dial are distinct, raise the front sight; look through the slit in the prism plate and bring the front sight in line with the forward station; when the needle comes to rest, read the azimuth through the prism.

COMPASS ERRORS.

164. The magnetic and the true meridian generally do not coincide. The angle between them at any point is called the **magnetic declination at that point.** If the needle points east of the true meridian, it is called an **east declination**; if west, a **west declination.** Magnetic declination varies in amount and direction at different points on the earth. The figure facing par. 166 (p. 80) **is a chart,** called **an isogonic chart,** which for the epoch gives, by curved lines connecting points of equal declinations, the approximate declination of points on the earth. **At no point is the declination constant.** It is subject to the following variations: **The daily variation** consists of a swing from the extreme easterly position at about 8 a. m. to the extreme westerly position about 1.30 p. m.; the mean position occurring about 10 a. m. and 5 p. m. The **daily variation** is from 5′ to 15′ of arc. **The secular variation** is a long slow swing, covering many years. In the United States all **east declinations** are now gradually decreasing and all **west declinations** gradually increasing at the rate of about 3′ per year. **The annual variation** is very small (less than 1′ per year) and need not be considered in surveying work. **The Lunar declination** is still smaller. All of the foregoing variations are periodic in character. **Irregular variations** due to so-called magnetic storms are uncertain in character and can not be predicted. Such variations are sometimes large. **Local attractions** may greatly disturb the needle, and often come from unknown sources. The

observer should have them constantly in mind and endeavor to keep all magnetic influences, such as magnetic bodies, electric wires, etc., at a distance from the instrument when the needle is being read.

The geometric axis of a needle may not coincide with its magentic axis, hence **the readings of two compasses at the same station may differ slightly.**

165. A simple way to detect—not measure—such disturbances is to take frequent back azimuths. If the position of the needle is normal at both stations, the azimuth and back azimuth will differ by 180°. If there is local attraction on the course, it will usually be stronger or cause a greater deflection at one station than at the other, and the azimuth and back azimuth will not differ by 180°.

Another way is, when taking the bearing to a station, to select a well-defined point beyond and on the same course. On arriving at the new station take a bearing from there to the selected point ahead. If it is the same as the first bearing to that point, there probably is no local disturbance. If the two bearings to the same point differ, there probably is local disturbance.

Corrections for abrupt deflections of the needle due to local attractions must not be distributed uniformly over the traverse. **A course in which local attraction is detected or suspected should be noted, and** if, on closing, an azimuth correction is necessary, it should be applied to the suspected courses.

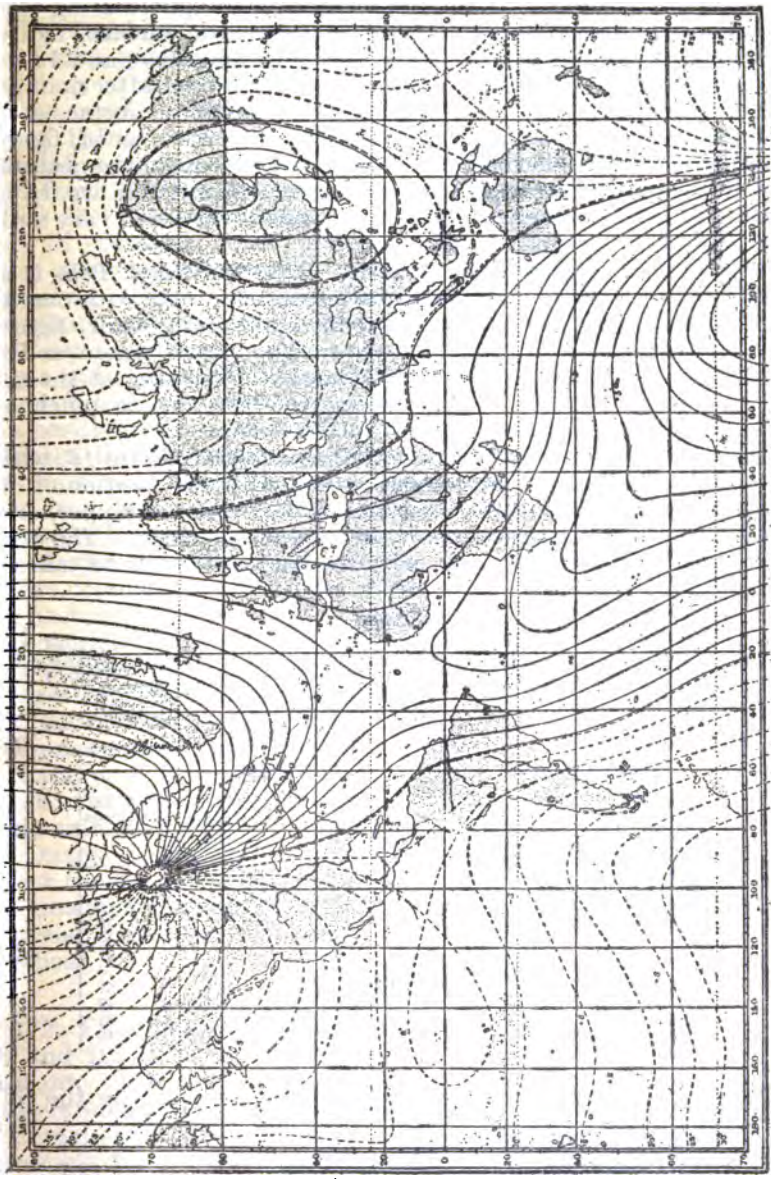
USE OF COMPASSES.

166. A good needle requires time to settle, even when the case is firmly supported, and the user should cultivate the knack of catching it at the middle of its swing, which is the desired reading. If the compass can be supported, it is always better to do so. Then the sight can be carefully taken and the position of the eye changed to read the needle. Wait till the swing gets down to 4° or 5°, which it will usually do in a few seconds. Then catch the highest and the lowest readings on the same swing and take their mean for the true reading. If the first swings are very large, catch the needle with the stop near the middle of the swing and release it quickly. This will suddenly check the swings and shorten the time in which the reading can be taken.

In using the box compass without a support, hold it sufficiently below the eye so that the swing of the needle can be seen. Point the line of sight in the required direction, catch the needle with the stop in the middle of the swing, and hold it stopped until the reading is taken. Stop readings are less accurate than sight readings, due to the difficulty in stopping the needle at the middle of the swing and to the tendency to displace the needle slightly in lifting it off the pivot. When the **stop is used, press it firmly and quickly.**

With the prismatic compass the stop is not used except to check the swings. Utilize a support if practicable. The prism having been adjusted for focus, level the case so as to bring the scale into focus, and when the swing becomes small read the extremes and take the mean.

APPROXIMATE ANNUAL CHANGE IN MINUTES OF ARC.



Lines of West Variation
" East "
indicates an annual increase
" decrease

58740°—18——6

TO DETERMINE THE DECLINATION OF THE COMPASS.

167. **First method; from the sun.**—Prick a small hole in a piece of tin or opaque paper and fix securely over the south edge of a table or other surface perfectly level, so that the sunlight coming through the hole will fall on a convenient place on the surface, figure *15*, The hole may be 2 feet above the table for long days and 18 inches for short ones. Half an hour before to half an hour after noon, mark the position of the spot of sunlight on the horizontal surface at equal time intervals of about 10 minutes. Draw a curve, as *bd*, figure *15*, through the points marked, and from point *c* in the horizontal surface and in a vertical line with the hole, *a*, sweep a circular arc, *ef*, intersecting *bd* in two points. The form of the curve *bd* will vary with the declination of the sun.

168. **Second method; from Polaris.—The true North Pole is about** 1.12′ distant from **Polaris** on a line joining that star with one in the handle of the dipper, and another in **Cassiopeia's chair**, figure *16*. One of these stars will always be above the horizon, wherever **Polaris** is visible. The polar distance of **Polaris** is decreasing at the rate of 19″ per year. It also varies during the year by as much as 1′. Both variations may be neglected in this work.

Imagine Polaris to be the center of a clock dial, figure *16*, with the line joining 12 and 6 o'clock vertical and with the position of one of the lines described (from Polaris to star in handle of dipper, or from Polaris to star in chair) as the hour hand of the clock. **The distance in angular distance of Polaris from the true north may be taken from the following table:**

TABLE I.

169. **Table showing the angular distances of Polaris** in different positions with respect to the pole. Epoch 1911; polar distance 70′. Latitude 0° to 18° north. This table may be used until 1930.

Clock reading of—		Angular distances.	Clock reading of—		Angular distances.
°Cass.	Z Ursae Maj.		°Cass.	Z Ursae Maj.	
		°			°
XII:30	VI:30	E 18	VI:30	XII30:	W 18
I	VII	35	VII	I	35
I:30	VII:30	49	VII:30	I:30	49
II	VIII	61	VIII	II	61
III	IX	70	IX	III	70
IIII	X	61	X	IIII	61
IIII:30	X:30	49	X:30	IIII:30	49
V	XI	35	XI	V	35
V:30	XI:30	18	XI:30	V:30	18

For higher latitudes multiply the tabular readings by the following:

Latitude 19° to 30°	1. 1
Latitude 31° to 37°	1. 2
Latitude 38° to 42°	1. 3
Latitude 43° to 46°	1. 4
Latitude 47° to 50°	1. 5
Latitude 51° to 53°	1. 6
Latitude 56° to 57°	1. 7
Latitude 58° to 59°	1. 8
Latitude 60° to 61°	1. 9

170. It is well to keep track of the position of Polaris by noting it frequently and taking the corresponding clock time. Then, if on a cloudy night a glimpse of Polaris is had, the observation may be taken, even though the other stars can not be seen.

171. **For practical details of the observation,** the following may serve as a guide: Select a clear space of level ground not too near buildings or any other object which might cause local disturbance of the needle. Drive a picket, leaving its top smooth and level, about 18 inches above the ground. Six feet north of the picket suspend a plumb line from a point high enough so that Polaris, seen from the top of the picket, will be near the top of the line, figure 17. The cord should be hard and smooth, about one-tenth inch in diameter. The weight at the bottom of the line should hang in a vessel of water or in a hole dug in the ground to lessen its vibration. Drive a second picket in range with the first one and the plumb line a short distance north of the latter. Make a peep sight by punching a hole one-tenth inch in diameter in a piece of paper and hold it on the top of the first picket; adjust it so that the star is behind the plumb line when looking through the peep. Note the position of one of the stars on the imaginary clock face at the moment the observation is taken. Mark the position of the peep on the top of the first picket and lay a straightedge or stretch a cord from that point touching the plumb line to the second picket. Place the north-and-south edge of the compass box against the cord or straightedge and read the needle.

The compass azimuth read is the **magnetic azimuth** to **Polaris** at the instant of observation. To find the **magnetic azimuth** of the **true meridian,** correct the compass reading by the angular distance of **Polaris** as given in **Table I, adding** the correction if **marked W;** subtracting, if **marked E. This method will give results true to within** one-fourth of a degree.

From an examination of the table it will be seen that when either S, Cass, or Z Ursae Major are at XII or VI, no angular distance is given. At these times, Polaris is on the true meridian and the magnetic azimuth to Polaris is the true azimuth. For a rough check on the magnetic declination, an observation on **Polaris,** taken when either the **dipper or Cassiopeia** is above the pole (near 12 o'clock on the imaginary clock dial) will give the **magnetic azimuth of the true meridian direct** to within less than the least reading of the ordinary compass.

THE SEXTANT.

172. This instrument is shown and its parts indicated in figures 18 and 19. The former is a very compact form, called the **pocket sextant.** The larger form, figure 19, has telescopes of different powers and also a telescope tube without lenses, which is used for reconnaissance work at short ranges. The pocket sextant has a telescope for use in astronomical and long-range terrestrial work. For ordinary reconnaissance and surveying, the pocket sextant is used without the telescope, the sight being taken through a small hole in a shutter which closes the telescope opening.

The adjustments are as follows: For the index glass, place the vernier at about 30° of the limb and examine the arc and its image in the index glass. If the arc and image appear continuous, the glass is in adjustment. If the image appears above the arc, the mirror

leans forward; if below, it leans backward. Adjust with screws if provided. or with slips of paper inserted between the mirror and its frame.

For the horizon glass.—Set at zero and observe a well-defined distant point, using the telescope. If the direct and reflected images coincide, the horizon glass is in adjustment. If not, adjust it until

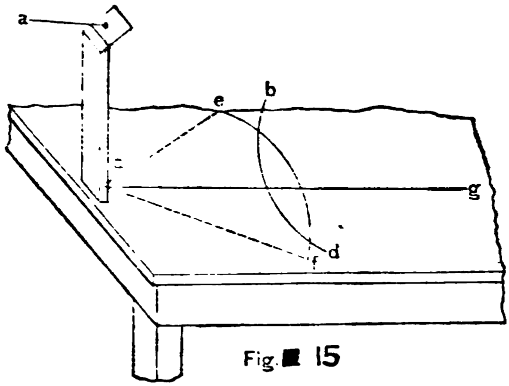


Fig. 15

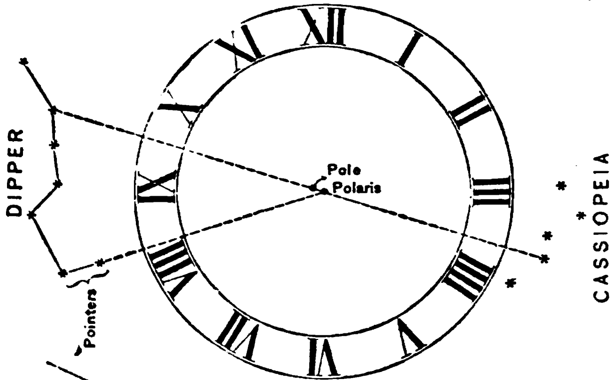


Fig. 16

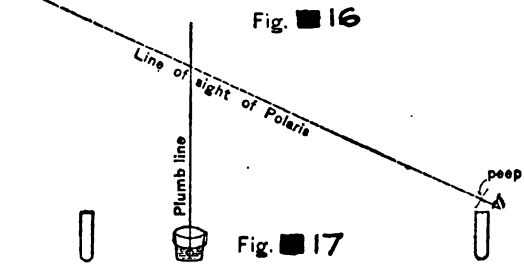


Fig. 17

they do, or if that can not be conveniently done, move the arm a short distance from zero until coincidence occurs. Read the vernier and apply that reading with its proper sign to all angles measured. Such a reading applied as a correction is called the **index error.** If

the index error is off the arc, that is, between zero and the end, it is additive. If on the arc, subtractive.

In the pocket form the horizontal glass only is adjustable. To adjust the pocket sextant, select a distant object with a clearly defined straight outline. Set the vernier carefully at the zero of the arc and look at the object through the peephole and the lower portion of the horizon glass. Turn the sextant about the line of sight

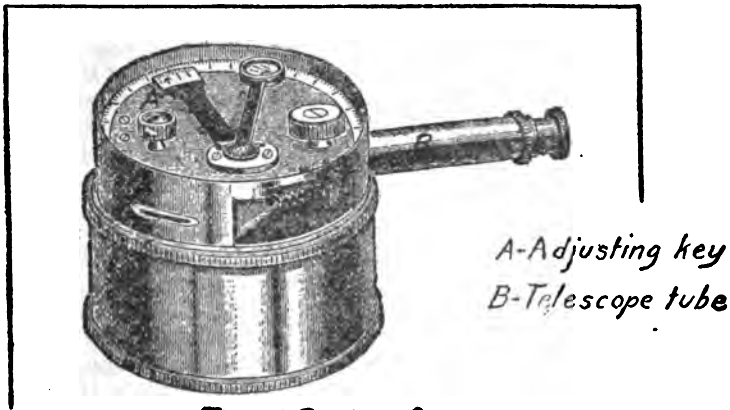


Fig. 18 Pocket Sextant

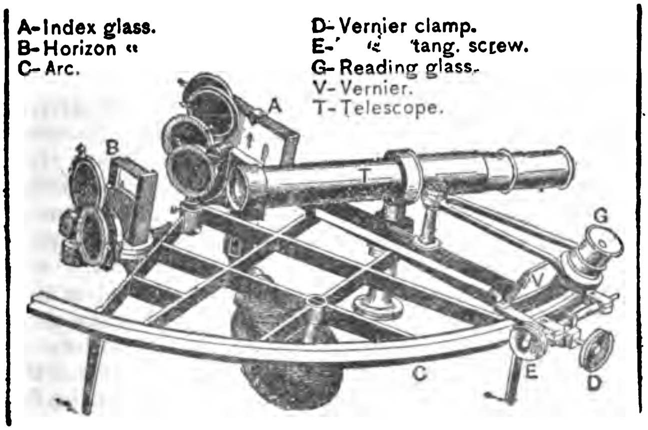


Fig 19

as an axis until the straight line appears to be perpendicular to the straight bottom edge of the horizon glass. If the instrument is not perfectly adjusted for this position, the straight line of the observed object will appear broken, in which case unscrew the smaller milled head A of the top plate, and using its small end as a key, turn the single adjusting screw in the cylindrical surface while looking at the object through the peep. The part of the image seen in the mirror will appear to move, and by turning the key in the

proper direction the two parts may be brought together. Next turn the sextant about 60° about the line of sight, and if the straight line again appears broken, use the key to slightly loosen one of the two adjusting screws in the top plate while looking through the instrument. If this brings the two parts nearer in line, the proper screw has been selected; if not, try the other one. Then turn the two adjusting screws in the top plate by corresponding amounts and in opposite directions and continue turning them alternately till the straight line becomes continuous. The two screws are opposed to each other, and care must be taken to use no considerable force and to always unscrew one before screwing up the other. When the adjustment is complete, the line should remain continuous and straight while the sextant is slowly revolved about the line of sight. If the index arm is then moved back and forth by turning the large milled head, the reflection of any object may be made to pass exactly over that object as seen through the clear glass.

For adjusting at night, screw the telescope in place. Pull its inner tube well out. Remove the sunglass from the eyepiece. Focus the telescope on a bright star by pushing in the tube till the image of the star is clear. Then, by turning the large milled head, make the star's reflected image pass through the field of view. If it does not pass exactly over the stationary image of the star, adjust the horizon glass with the two screws in the top plate till one image will pass exactly over the other. Next set the vernier accurately to the zero of the arc, and with the single adjusting screw in the cylindrical surface make the two images appear as one. The instrument is then completely adjusted. The daylight method is most convenient, but it is well to test the adjustment by the star method before attempting to do any astronomical work.

In the cylindrical surface just below the zero degree end of the arc are two projecting levers which move colored glasses to be used in looking at the sun. At other times these glasses should be depressed through the opening in the bottom plate by first sliding the brass stud in the plate and then pushing the two levers. The telescope also has a colored sun glass secured on the eye end which must be removed when observing any other object.

Adjustment of the line of sight.—Two parallel wires are placed in the focus of the objective of the telescope, the middle point between which marks the center of the field of view. The line joining this point and the optical center of the objective is the axis of the telescope or the **line of sight.** This line should be parallel to the frame of the instrument. To test the adjustment, turn the telescope in its collar until the wires are parallel to the frame. Select two objects which are at a considerable distance apart, as the sun and moon when distant 100° or more from each other. Point the telescope to the moon and bring the image of the sun tangent to it on one of the wires. Move the instrument until the images appear on the other wire. If they are still tangent, the telescope is adjusted; if otherwise the adjustment is made by two screws in the collar, loosening one and tightening the other. In some instruments the adjustment of the parallelism being supposed to be carefully attended to by the makers, the screws are wanting. With a properly adjusted instrument, two images seen in contact on the wires will overlap in the center

of the field. If the two images are tangent on the lower wire and appear to separate on the wire farthest from the frame, the object end of the telescope droops toward the frame.

ERRORS OF THE SEXTANT.

173. To whatever division of the arc the index may point when the mirrors are parallel, this division is the piont of beginning of all angle measurements. In other words, it is the temporary zero, and the difference in arc between this temporary zero and the actual zero of the arc is the index error. Due to unequal expansion and contraction, this index error will not remain the same. It should, therefore, be determined anew each time the instrument is to be used. To measure it, bring the mirror to parallelism by producing a perfect coincidence of the direct and reflected images of a distant point or star; read the vernier, giving the result the proper sign—**minus** if coincidence occurs when the index is on the same side of zero as the greater part of the arc; **plus** if on the same side of the small portion of the arc, called "off the arc." Another error which must be looked out for, is that due to "**eccentricity.**" This error is caused either by an original defect in the instrument or by a bending of the frame by varying temperatures or by accidental blows.

To determine this error, measure with a transit the angle between two distant points having the same elevation. Make several readings of the same angle with the sextant and take the mean. The difference between the transit determination and the mean of the sextant determination, will be the effect of the eccentricity for **that particular reading of the sextant.** The operation should be repeated for the whole arc at short angular distances and the results tabulated. From time to time this tabulation should be checked to see that no change has occurred.

USE OF THE SEXTANT.

174. When angles between terrestrial objects are to be taken with the sextant, the index is set at zero, and holding the instrument in the right hand so that the plane of the frame coincides with the plane through the objects to be observed, with the telescope on the upper side if the angle is approximately horizontal and on the left side if the angle is vertical, sight the left-hand object. There will be a slight lack of coincidence in the two images due to parallax, even if the instrument has been adjusted for index error by sighting at a star. Move the index arm until there is coincidence and read the vernier. Use this reading as index error. Now keeping the left-hand object in the field by sighting through the transparent part of the horizon glass, move the index arm with the left hand until the other object appears in the mirror portion of the horizon glass opposite the first point. Bring the second point exactly opposite by the tangent screw. Test the coincidence of the images by twisting the instrument so as to make the reflected image move back and forth across the direct image. **Read the vernier and apply index and eccentricity correction.** In rapid work the telescope is not used; sight is taken through the telescope ring. Make it a rule to commence taking angles from the object farthest to the left, then from the next farthest, and so on, always working from left to right. Avoid very large or very

small angles. Though the angles measured with the sextant are seldom horizontal angles, it is usual to plot them as such in filling in a topographical or hydrographic survey. The errors due to obliquity will be nondiscernible in work plotted with the ordinary protractor.

THE ENGINEER'S TRANSIT.

175. This instrument is shown, and the names of its parts indicated in figure 20.

To set up the transit.—Place the tripod with the legs extended far enough to give a stable base and so as to make the top surface of the

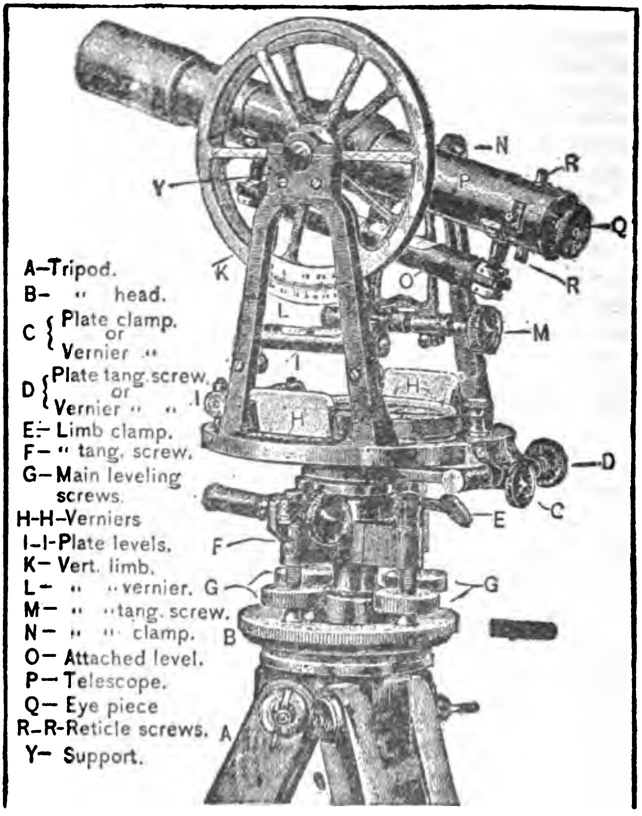


FIG. 20.

tripod head horizontal or nearly so. On level ground the legs will be equally extended. On inclined ground, two legs on the lower side should be on the same level and relatively close together. The third leg is moved straight uphill at right angles to the line of the lower two; the amount this leg is thrown uphill is that sufficient to bring the tripod head roughly level. If the instrument has not already been screwed to the tripod, remove the tripod cap and screw on the instrument in its place. Hang the plumb line on the hook (depending through the tripod head). Level the instrument as follows: Unclamp the vernier plate and turn the transit so that one

of the plate levels is parallel to one pair of leveling screws. The other plate level will be parallel to the other pair. Bring the bubbles of the levels to the center in succession by means of the leveling screws. Always turn one of a pair down as the opposite one is turned up and avoid more pressure of the screws against the plate than is necessary for a firm bearing. If a screw turns hard at any time it is either sprung or has been set up too tight. In turning a pair of leveling screws always move the thumbs toward each other or away from each other. The bubble will follow the motion of the left thumb. **If the screws are too tight, unscrew either but not both.**

With the level bubbles in the centers of their tubes, the plate will be level if the bubbles are in adjustment. Turn the transit slowly in azimuth and watch the bubbles. If they remain in the centers, the plate is level and the levels are also correct. If either bubble leaves the center, the amount of its motion indicates the amount by which it is out of adjustment. If the amount is small, it may be neglected; if large, the adjustment should be made as hereafter described. For short lines the level error may be neglected if the entire bubble remains in sight during the entire revolution. Adjust the leveling screws in this case so that the travel of the bubble will be equal on both sides of the center.

Parallax.—Having leveled the instrument, point the telescope at the sky. Focus the eyepiece until the cross hairs appear sharply distinct. This should eliminate parallax. To test the adjustment, point the telescope at some terrestrial object and bring it to a proper focus by means of the focusing screw of the object glass. Bring the intersection of the cross hairs on some well-defined point of the image. Now move the head laterally, watching the intersection. If there is no relative motion of the cross hairs and the image, parallax has been eliminated. The eyepiece once adjusted for parallax need not again be focused for the same observer unless it has been disturbed. The test, however, should be occasionally repeated. **The object glass must be focused for each separate sight.** The instrument is now ready for use or adjustment.

The adjustments of the transit are: (1) To make the plane of the plate bubbles truly perpendicular to the vertical axis of the instrument; (2) to make the line of sight truly perpendicular to the horizontal axis; (3) to make the horizontal axis of the telescope truly perpendicular to the vertical axis of the instrument. These three adjustments are made to depend on the principle of reversion, the effect of an error being doubled by a reversal of the instrument. **The adjustments should always be made in the order given.**

(1) **Adjustment of the bubble tubes.**—One level tube is adjusted at a time. **Clamp the lower limb.** Bring the bubble in the center of its tube with the leveling screws. Revolve the vernier plate 180°. If the bubble axis is not truly perpendicular to the axis of revolution, the error will be indicated by the bubble leaving the center of the tube. **The movement of the bubble measures double the error.** Correct the error by bringing the bubble (by means of the small capstan screws on the tube) **halfway** back to the center. If it is brought exactly halfway back, the error is eradicated. Verify the adjustment by recentering the bubble, as before, with the leveling screws and revolving 180°. If the bubble again leaves the center, there is some error remaining. Correct this residual error as before and verify.

Two attempts should be sufficient to correct all the error. Adjust the other bubble tube in the same manner. **While adjusting one bubble see that the other is centered.**

(2) **Adjustment of the line of sight.—Clamp the lower limb.** First make the vertical hair perpendicular to the horizontal axis. To do this sight the vertical cross hair on some well-defined point, clamp both plates, rotate telescope about horizontal axis. If point does not appear to travel along vertical hair, loosen screws (R) holding cross hair ring, and by lightly tapping on one screw rotate ring until above condition is fulfilled. **Tighten screws and proceed with second part of adjustment as follows:** See that lower limb is clamped, unclamp the upper or vernier plate. Direct the intersection of the cross hairs at a sharply defined point A, 200 or 300 feet away; clamp the vernier plate, then plunge the telescope (revolve on its horizontal axis), and have an assistant set a point B (a marking pin or a pencil mark on a vertical wall) in line with the intersection of the cross hairs and at approximately the same distance away (points A and B should be at about the same elevation), but in the opposite direction. Unclamp the vernier plate and revolve the telescope in azimuth (about the vertical axis) until the intersection of the cross hairs is again accurately on point A; clamp the vernier plate, and again plunge the telescope and set a point C in line with the intersection of the cross hairs and beside the point B. The distance between the points B and C is four times the error. Mark a point D one-fourth of the distance between B and C, measured from C. Move the cross-hair ring by loosening the reticle screw on one side of the telescope and tightening the one on the opposite side until D is at the intersection of the cross hairs. To verify, repeat the whole operation. Two attempts should be sufficient for accurate adjustment.

(3) **Adjustment of the horizontal axis.**—The plate bubbles being truly perpendicular to the vertical axis of revolution and the line of sight being truly perpendicular to the axis of the trunnions, set up and level the transit. Clamp the lower limb and release the vernier plate. Now with the cross hairs bisect some sharply defined point A, at a very high angle (gable of a house near by); clamp the vernier plate. Depress the telescope and mark down, at a convenient point under the point A and at about the level of the telescope, the point B in line of sight. Now unclamp the vernier plate and revolve the instrument in azimuth (about vertical axis), plunge the telescope (revolve about horizontal axis), and sight at A. The telescope is now inverted (bubble up). Depress the telescope and set a point C in the line of sight and beside the point B. The distance between B and C measures twice the error. Correct for **one-half** the error by the adjusting screw underneath one end of the horizontal axis.

Adjustment of the telescope level.—If there is a level attached to the telescope, it may be adjusted by the "peg" method after the other adjustments are made, as follows: Set up midway between two stakes, which have their tops at about the same elevation, level the transit, and with the bubble of the attached level at the center read a rod on each stake The difference in the readings is the **true difference** in level of the tops of the stakes. Move the instrument toward one of the stakes, and set it up so that the eyepiece is about over the center of the stake. Place the rod on the stake near the eyepiece, and set the target in the middle of the field as seen through the

object glass. Set up the rod on the far stake with a target set at the reading just taken through the object glass, plus or minus the difference of level between stakes—plus if lower, minus if higher. Bisect the target with the cross wires. **The line of sight must now be horizontal,** and, keeping the vertical motion clamped so as to retain the pointing, adjust the bubble of the attached level to the center by means of the small screws at the movable end of its tube. Both line of sight and axis of bubble are now horizontal, and therefore parallel.

Note that the position of the intersection of the cross wires in the field is a matter of convenience mainly. It is best to have it near the middle of the field, and it can be placed there by inspection with all needful precision before making the adjustment of the line of sight.

Vertical circle adjustment.—While the line of sight and attached bubble are still horizontal, the screws holding the vernier for the vertical arc should be loosened and the vernier moved until the reading is **zero.** If the vernier is not adjustable, the reading of the vernier when the attached level and line of sight are horizontal may be taken as the index error and applied to all readings (or the line of sight may be adjusted to the vernier when reading zero; this will involve a retest of all previous adjustments).

An instrument may at times appear to be out of adjustment because some part is loose.—The object glass may be partly unscrewed or an adjusting screw may be only partly tightened. Level bubbles or cross wires occasionally become loosened; therefore, before commencing the adjustment of an instrument, look out for such defects. When it is thought that an adjustment has been completed, always test the instrument before using. All adjusting screws should be screwed tight enough to hold, and yet not so tight as to injure the threads or put a severe strain on any other part. Especial care should be taken not to strain the cross-wire screws.

To eliminate effects of errors in adjustment, the instrument should be used as follows: To avoid errors in plate bubbles, level up, turn 180° in azimuth, and bring bubbles halfway back by means of leveling screws. This makes vertical axis truly vertical, and the bubbles should remain in the same parts of their respective tubes as the instrument is turned about the vertical axis. Errors in the line of sight and horizontal axis are avoided by using the instrument with its telescope direct, and then in its reversed position and taking the mean of the results, whether the work is running lines or measuring angles. Errors of eccentricity are eliminated by taking the mean of the reading of two opposite verniers. Errors of graduation are nearly eliminated by reading the angle in different parts of the circle, or by measuring the angle by repetition. **Where only one vernier is read in determining an angle, always read the same one.**

USE OF THE TRANSIT.

176. To measure a horizontal angle, set up over the vertex of the angle to be measured and direct the telescope along one of the sides of the angle. Clamp limb and plate—if the latter is set at zero it is more convenient—and with the tangent screw of the limb bring the intersection of the cross hairs on a definite point of the line. Read each of the two verniers and record, calling one vernier **A** and one **B.**

Unclamp the plate—not the limb—and direct the telescope along the other line. Clamp and bring the cross hairs to a definite point with the vernier tangent screw. Read and record as before. Take the differences of the two readings **A** and **B**, respectively. If these differences are the same, it is the value of the angle. If not, take the mean of the differences as the value. **For greater accuracy,** the method of **repetition** is used. After the first measurement is made, unclamp the limb—not the plate—and resight on the first point by means of the limb tangent screw and proceed as before. The reading of the vernier is now twice the angle. Continue the repetitions until the desired number are made. The last reading divided by the number of measurements is the value of the angle. To guard against errors, it is well to read and record after each measurement.

To measure a vertical angle.—Point the instrument, clamp the horizontal motions, and make the readings on the vertical limb. For greater accuracy when there is a complete vertical circle, revolve the instrument through 180°, plunge the telescope, and take new readings. If the results differ, use the mean.

To run out a straight line.—Set up accurately over the initial point. Point the telescope in the required direction and establish a second point. These two determine the line which is to be run out. Set up over the forward or second point; lay the telescope on the initial point; clamp limb and plate, plunge telescope, and set a point forward. If the adjustments are good, this third point will be in line with the first and second and the line may be prolonged by repeating the steps taken at the second point.

If the adjustments are not good, set a third point as before. Then unclamp the limb and turn 180° in azimuth and lay on the initial point. Clamp and plunge again and set another third point beside the first one. Take the middle point between the two for the true third point. This method eliminates errors of adjustment, except those of the plate levels. These are so easily observed and corrected that they should never exist when close work is required.

TRAVERSING.

177. The transit must be set at each station with a 0–180° line of the azimuth circle parallel to its position at preceding stations. This is called **carrying an azimuth.** The direction chosen for the 0–180° line is usually the true N. and S., or as near it as data at hand will permit.

Having observed the second station from the first, proceed to the second, set up, and set one of the verniers at its reading from the first to the second station, plus 180°, or at the **back azimuth.** Point at the first station and clamp the limb. The line 0–180° is now in a position parallel to that at the first station. Unclamp the plate, direct the telescope to the third station, and proceed as before.

Also see paragraph 135.

A VERNIER.

178. A vernier is an auxiliary scale by means of which the principal scale can be read more closely than can be shown by actual subdivisions on the principal scale.

Consider **AB**, figure 21, as part of a scale of equal parts. Construct the auxiliary scale or vernier **CD**, the total length of which is equal

to 9 of the smallest divisions of the principal scale, but divided into 10 equal parts instead of 9, which makes each division of the vernier $\frac{9}{10}$ the length of the division of the scale.

When the zero division of the vernier, indicated by an arrow, is coincident with a division, as 31, of the scale, the reading is 31 and it is obvious that the first division of the vernier is to the left of 32 in the scale by $\frac{1}{10}$ of the distance between 31 and 32. Similarly, the second, third, etc., division of the vernier is 2, 3, etc., tenths to the left of the 33, 34, etc., division of the scale. To make any division of the vernier, as 2d, 3d, 5th, or 8th, coincide with the division of the scale next ahead of it the vernier must be moved to the right 2, 3, 5, or 8 tenths of the length of one division of the scale, and the arrow will then be opposite a point on the scale 2, 3, 5, or 8 tenths of the distance from 31 to 32, or at 31.2, 31.3, 31.5, or 31.8. The quantity obtained by dividing the value of one division of the scale by the number of divisions of the vernier is called the **least count of the vernier.** Only one intermediate vernier division can coincide with a scale division at the same time and the number of the coincident vernier division, counting from the arrowhead, is the number of times the least count must be added to the last scale division passed by the arrow to get the true reading.

To read any vernier note the value of the last scale division passed by the zero of the vernier and to it add the least count multiplied by the number of the coincident vernier division.

Mistakes will be avoided and the reading facilitated by estimating in advance the fractional part of the division of the principal scale.

A vernier constructed as described is always read ahead of the zero, or in the direction in which the scale graduations increase, and is called a **direct vernier.** Verniers may also be constructed by dividing the length of a certain number of divisions of the scale, as 11, into equal parts one less in number, as 10. The principles of operation and method of reading are the same, except that the coincident line is to be found behind the zero of the vernier, or in the direction in which scale graduations decrease. This form is called **retrograde.** It is but little used.

If the scale is graduated in both directions, as is often the case, the vernier is doubled, the zero in the middle and each side forming a direct vernier for the graduations increasing in the same direction. This form is called **double direct,** figure 22. The most compact form is that shown in figure 25, called the **folded vernier,** in which the graduations are numbered from the middle to one end and continue from the other end to the middle. This is read as a direct vernier in either direction. If the coincident line is *ahead* of the middle or in the direction of *increasing graduation,* take its number from the middle as zero. If it is *behind* the middle, or in the direction of *decreasing graduation,* take its number from the nearest end, counting the end line as numbered on the vernier.

Verniers are also constructed on cylindrical surfaces and on conical surfaces. The principles and method of reading are the same for all.

179. **The plane table** is shown and its parts indicated in figure 26. The adjustments of the instrument are entirely analagous to those of the transit. The plate levels are carried either on the alidade or on the declinator. In reversing for level tube adjustment, care must be taken to have the alidade or the declinator, as the case may be,

cover the same part of the board in both positions by marking tw corners on the paper by faint pencil lines.

To set up the plane table over a known station.—In the followin discussion, it is assumed that a number of stations, the locations which have been secured by transit triangulation, have been plott on the plane table sheet as will usually be the case in plane table wor Theoretically before any work can be done on the drawing from a station, the instrument must be so set up that the plotted position the point is vertically over the corresponding station on the groun that the board must be truly horizontal; and that the meridian (tru

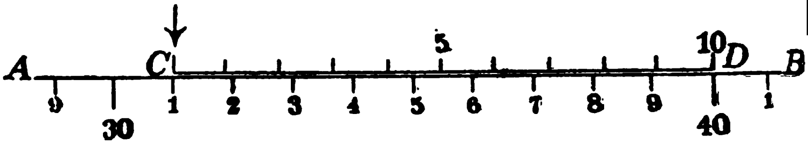


Fig. 21.

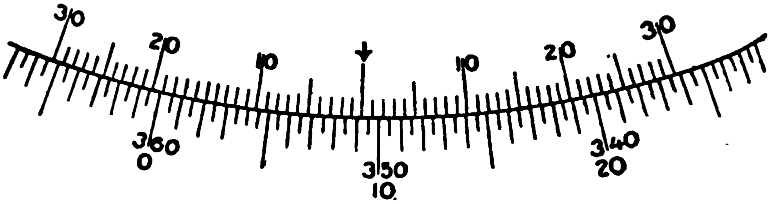


Fig. 22.

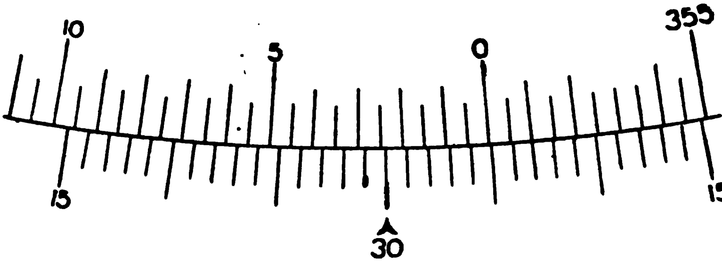


Fig. 23.

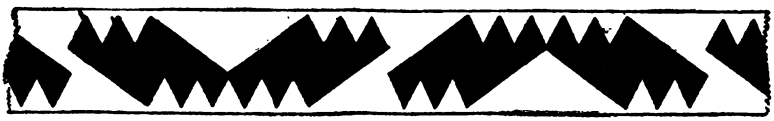


Fig. 24.

or magnetic) of the point on the paper must lie in the plane of the meridian of the corresponding station. In the figure, there is shown a devise for plumbing any point on the paper over the corresponding point on the ground. Such a refinement is necessary only in very close work on a very large scale. For maps on a scale of 6 inches to the mile or smaller it is sufficient to place the point over the station by the eye.

The board is leveled by the leveling screws or as in some instruments by manipulation of the board by the hands, the board being mounted on the tripod by a ball-and-socket joint. Wilson says that the inclination of the board from the true horizontal plane or the

amount which it is out of level affects the location in azimuth far less than would be at first estimated. For an inclination of 15° the azimuth is affected only 1°. For an inclination of about 3°, the error in azimuth would amount to about less than 2½ minutes (about 4 feet in a mile). An undue amount of time should not, therefore, be spent in leveling when orientation is to be secured from nearby stations and short side shots only are to be taken.

The third requirement for a proper set up is met by **orienting** the board, after it has been set over the station and leveled. By orienting is meant the adjustment of the board in azimuth so that the line from the station point to any other point shall be parallel to the corresponding line between the two stations in nature. To orient the board, therefore, place the edge of the ruler of the alidade on the station point and any other distant point. Unclamp and swing the board in azimuth until the line of sight of the alidade intersects the other station. Check by a sight on another station or so.

If the magnetic meridian has not been plotted on the paper, place the declinator on the board, after orientation, and allow the needle to come to a rest. Draw a line the full length of the declinator side and mark the north end. The magnetic meridian will assist in orienting the board when plotted stations can not be seen and will always be valuable as a check.

Locations by intersection.—The plane table finds its greatest use in quickly securing a secondary control for topographical work. In open, hilly country, this control is most rapidly made by locating a number of points by intersection and resection. Points are located by intersection by **setting up the plane table and orienting** as described above over a known station and drawing rays to natural or artificial signals, being careful to number each ray and note in a notebook the object sighted by each numbered ray. A second known point is then visited, the plane table set up and oriented, and a series of rays to the same signals drawn. The intersection of corresponding rays locates the signals.

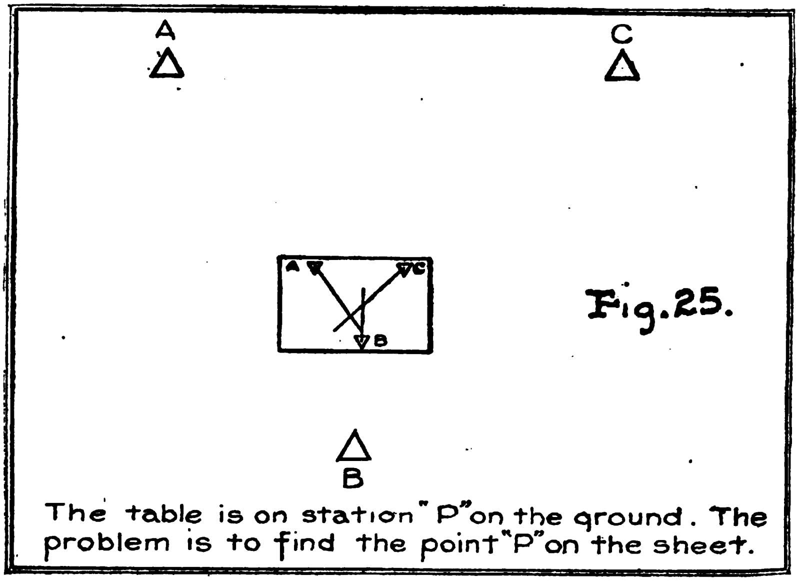
All stations which are important in the propagation of the plane table triangulation, should be checked by a third ray drawn from a third known station. Where it is difficult to get a third intersection, locations by two rays will serve for tertiary points of control. But if such a station is occupied by the instrument for propagation of the triangulation, the location and orientation should be checked by **resection methods.** It is sometimes desirable to locate a plane table station from a line only one end of which can be occupied by the table. Let A and B represent the points on the ground at the ends of the line: C is the signal which is to be located, and *ab* represents the line plotted on the plane table sheet. Set up at A, the end of the line which is accessible, and orient the table by sighting B with the alidade along *ab*. Then, centering the alidade on *a*, draw an indefinite line toward C. This line should be drawn the full length of the alidade. The table is then taken to C and oriented by means of the line just drawn. Since the position of *c* on the indefinite line is now known it is necessary to estimate its position on the map and to use this point in setting the table over C. If the alidade is now centered on *b* and sighted toward B, a resection line may be drawn, and this line will cut the first indefinite line thus locating the point *c* desired. The position of *c* found by this method should be

checked if possible by resection lines from other points whose positions are known to be correct.

THE THREE-POINT PROBLEM.

180. The plane table may be set up at any place where three triangulation points (plotted on the sheet) can be seen and the position of this plane table station can be determined and plotted on the sheet by observations from this point. The following is one of the graphical solutions:

If three signals A, B, and C have their plotted positions at *a*, *b*, and *c*, and if the table be set up at any point and oriented correctly, the resection lines drawn from *a*, *b*, and *c* will pass through *d*, the plotted position of P. Since there is no means of accurately orienting the table, the position of *p* being unknown at the start the table



must be oriented approximately by the compass. If the place table is not oriented exactly, the three resection lines will not ordinarily pass through a common point but will form a triangle known as the triangle of error (fig. 25). From this triangle of error the true position of *p* may be estimated, and by a second trial a new triangle of error may be obtained which is smaller than the former. By successive trials the triangle may be made so small that it is almost a point. In practice very few trials are necessary, the triangle often being reduced to a point in the second trial.

If the table is on the circumference of the circle through the three points, its position is indeterminate. When point *p* is inside the triangle ABC it is in a favorable position for an accurate location. If the table is outside this triangle there are certain positions of the signals which are not favorable, especially when the angles subtended by the sides of the triangle formed by the signals are small and the

middle signal is farthest from *p*, but if the middle signal is near *p* the location of *p* is good.

If P lies inside the triangle ABC then *p* will lie inside the triangle of error and vice versa. If a circle is passed through *a*, *b* and the intersection of the resection lines from *a* and *b* it will pass through the true position of *p*; similarly a circle through *b*, *c* and the intersection of the resection lines from *b* and *c* may be sketched and in this manner a close estimate of the position of *p* made for the second trial. In practice, the circles are not actually constructed. A small portion of the arc of each circle is sketched, so as to indicate the position of *p*, preparatory to a second trial. The correct distance of *p* from any resection line is proportional to the distance of the plane table from the signal from which that line was drawn.

Tracing-paper solution of the three-point problem. By the use of tracing paper the three-point problem is solved approximately with great rapidity. Setting up the table on the unknown point P, fasten on it a piece of tracing paper of sufficient size to include the positions of all four points. A fine point is marked upon the tracing paper to represent the position of *p*. The alidade is then centered about the point *p* and pointed successively at the three known points A, B, C, and the lines *pa*, *pb*, and *pc* are drawn on the tracing paper. The alidade being then removed and the tracing paper released. The latter is then so shifted over the plane table sheet that the line *pa* shall pass through the located point *a*, the line *pb* through *b*, and the line *pc* through *c*. Then with all three lines passing through the known points, the point *p* is exactly over its correct position on the plane table paper and may be pricked through to the latter. The point *p* is now located and **orientation** is secured by placing the alidade over *p* and any other known point and moving the table in azimuth until the other point is intersected by the line of sight. Check by a sight on two other points. It will usually be found that there is a slight error in the location of *p*. In case the location of the point is desired more accurately, draw rays from each of the three known stations, and proceed as in the solution given next above.

RANGING IN AND LYING IN.

181. It will often be desirable to locate the table at a point from which only two known points are visible. This may be done by ranging in or lining in or by use of the magnetic meridian and needle. This method depends upon the fact that if the table **can be oriented in any way** the location of the point where the table is set up can be found by resection rays drawn from any two stations in view or by resection from one point, if station is on a plotted line. Therefore if any line has been drawn on the place table sheet which represents a physical line on the ground, as a straight fence, a railroad tangent, road tangent, etc., the plane table can be set up at any point along the fence or on the tangent and oriented by laying the alidade along the line on the paper and turning the table until the paper line is parallel to the physical line represented. As soon as the table is satisfactorily oriented, pivot the alidade over any known signal and draw a resection ray from that station. The intersection of this resection line and the line of the fence or tangent is the location of the instrument. Similarly it will often be possible to set up the instrument on

the line through two stations and orient by sighting along the line, locating by a resection line from a third station. Or, set up and orient by the needle and draw two resection rays from two known points. The intersection of these will be the location of the table. These methods are satisfactory only for the location of station from which local topography is to be secured. They are not good for extension of triangulation.

PLANE TABLE TRAVERSES.

182. Once the plane table has been set up and properly oriented over any plotted point, any other point in the vicinity may be located by drawing a ray through the station in the direction of the point and laying off to scale on the ray the distance to the point as ascertained by stadia or other method of measurement.

A second point so located on the sheet may, of course, serve as an instrument station, and if the plane table be moved to the second point it may be set up with the plotted position of the point over the actual point on the ground and the instrument oriented by placing the alidade along the ray and moving the table in azimuth until the first point is "backsighted" by the line of sight. With the instrument oriented, a third station may be located from the second in the same manner and by thus proceeding from point to point **a plane table traverse** may be run and plotted more quickly than can be done by transit and stadia and computation by latitudes and longitudes. As with all other traverses of any considerable length, plane table traverse should not be made a part of the map until they have been adjusted to locations made by more accurate methods. The graphical adjustment described in paragraph 148 is very convenient for the adjustment of plane table traverses. In very large scale work, back flags must be used in order to secure correct orientation and the orientation of the table should be checked whenever possible on triangulation stations. In work on smaller scales better results will be secured if orientation is made by magnetic needle instead of backsights. Wherever local attraction is suspected, backsights should be taken in this work to check the needle orientation.

DETERMINATION OF DISTANCES.

183. Distances passed over may be determined by the stride of man or horse, by the time taken by a rated horse, by the revolutions of a wheel, by chain or tape, and by stadia. Distances which are not passed over may be determined by estimation, stadia, or by intersection.

REDUCTION TO THE HORIZONTAL.

184. Distances measured along a slope may require a correction before plotting them on a map, as all map distances are, or are supposed to be, measured in a horizontal plane. Such corrections, when made, are called **reduction to the horizontal.** The following table gives horizontal distances corresponding to sloping distances for gradients up to 30°. The correction for slopes of 6° and less is too small to be plotted on the customary scales and is usually neglected. In flat or ordinary rolling country, the correction will rarely be necessary.

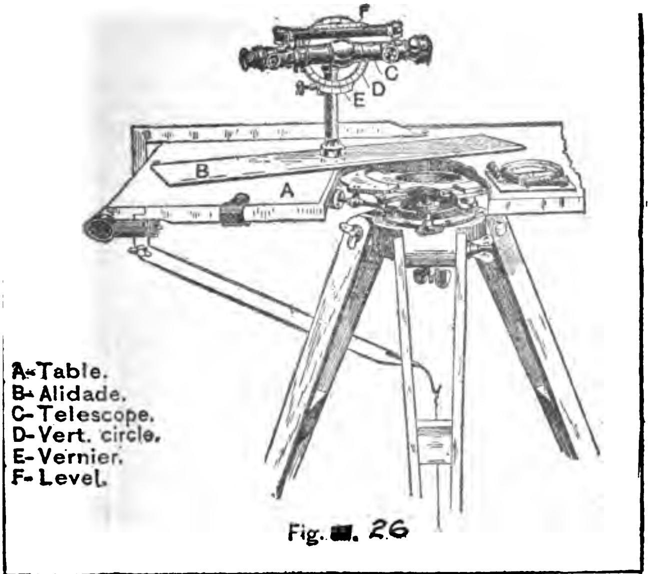


Fig. 26

Plane Table

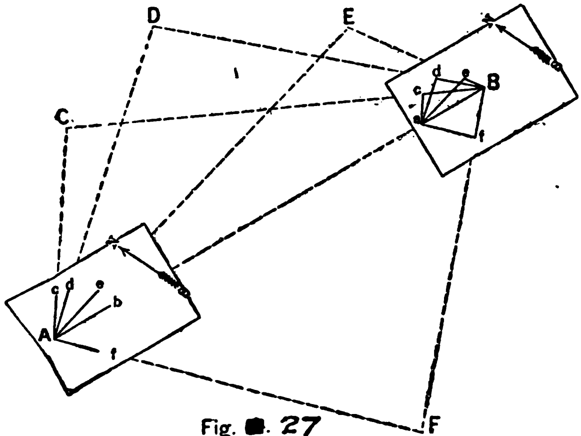


Fig. 27

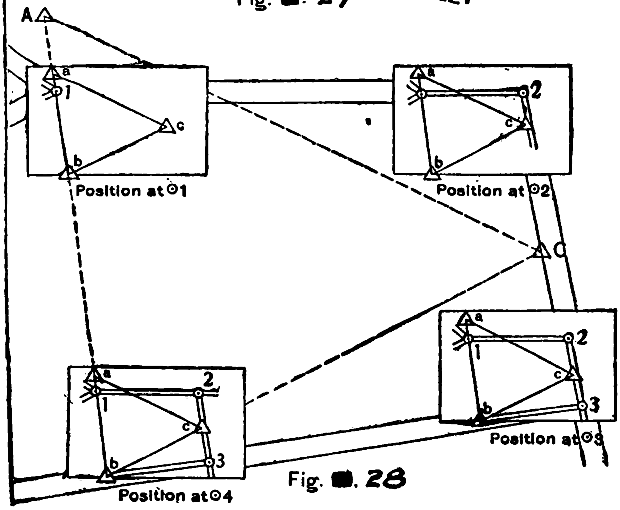


Fig. 28

Traversing and Intersecting by Plane Table

TABLE II.

185. **Horizontal distances** for gradients of 0° to 30° corresponding to distances on the slope:

Gradient in degrees.	Horizontal distances for sloping distances of—								
	1	2	3	4	5	6	7	8	9
1	09998	19997	29995	39994	49992	59991	69989	79988	89986
2	09994	19988	29982	39976	49969	59963	59957	79951	89945
3	09986	19972	29959	39945	49931	59918	69904	79890	89877
4	09976	19951	29927	39902	49878	59854	69829	79805	89781
5	09962	19924	29886	39848	49810	59772	69733	79695	89657
6	09945	19890	29836	39781	49726	59671	69616	79562	89507
7	09925	19851	29776	39702	49627	59553	69478	79404	89329
8	09903	19805	29708	39611	49513	59416	69319	79221	89124
9	09877	19754	29631	39507	49384	59261	69138	79015	88892
10	09848	19696	29544	39392	49240	59088	68936	78785	88633
12	09781	19563	29344	39126	48907	58689	68470	78252	88033
14	09703	19406	29108	38812	48515	58218	67921	77624	87326
16	09613	19225	28838	38450	48063	57676	67288	76901	86513
18	09510	19021	28532	38042	47553	57063	66574	76084	85595
20	09397	18794	28191	37588	46985	56381	65778	75175	84572
22	09272	18544	27815	37087	46359	55631	64903	74175	83446
24	09135	18271	27406	36542	45677	54813	63948	73084	82219
25	09063	18126	27189	36252	45315	54378	63441	72505	81568
26	08988	17976	26964	35952	44940	53928	62915	71903	80891
27	08910	17820	26730	35640	44550	53460	62370	71280	80190
28	08829	17659	26488	35318	44147	52977	61806	70636	79465
29	08746	17492	26238	34985	43731	52477	61223	69969	78716
30	08660	17320	25981	34641	43301	51961	60622	69282	77942

The horizontal distance corresponding to *any* sloping distance and *any* angle or gradient may be found by multiplying the sloping distance by the **cosine** of the angle.

MEASURING DISTANCES WITH STEEL TAPE.

186. The steel tape furnishes a convenient, rapid, and economical means of measuring any distance for any desired degree of accuracy up to about 1 in 300,000. For topographical surveying a length of 100 feet is most convenient. For base-line measurement the length should be from 300 to 500 feet and its cross section from two to three one-thousandths of a square inch.

The length of a topographical base will depend on the stretch of level or evenly sloping ground available. Such bases have varied in length from ½ mile to 3 or 4 miles. **It is more important to be able to extend the base by well-conditioned triangles** than to measure a long base. The ideal site is on level, even ground, from which good views of the surrounding country can be obtained.

MEASUREMENT OF BASE.

187. A topographical base should always be measured with a steel tape. This should be carefully compared with a standardized steel tape before and after using. A standardized steel tape is one where exact length for a given temperature pull as well as its coefficient of expansion has been determined by the National Bureau of Standards at Washington, D. C. Any steel tape may be standardized by sending it to that bureau. The manufacturers for a slight extra cost will furnish a certificate giving the temperature and tension at which a tape agrees with their standard (facsimile of that in the national bureau) or they will furnish the tape with the certificate of test made by the national bureau itself. The standardized tape itself should not be used in the field lest it be broken.

For an accuracy of 1 in 5,000 the tape may be used in all kinds of weather, held and stretched by hand, the horizontal position and

amount of pull estimated by the chainman. The temperature may be estimated or read from a thermometer carried for the purpose. On uneven ground the end marks are given by plumb line.

FOR AN ACCURACY OF 1 IN 10,000 TO 1 IN 50,000.

188. The line of the base should first be cleared of trees, bushes, or high grass; small mounds, etc., should be removed.

Each terminal point of the base should, if time allows, be marked by burying a bottle 2 feet underground; accurately centered over this should be built a small masonry pillar, surface flush with the ground. A fine mark should be made in a metal plug sunk into the surface of the pillar.

Having marked the terminal points, set a transit over one of them and direct it on the other, over which a pole should have been fixed; with the transit align stakes at intervals of about 300 yards. By means of this alignment hammer into the ground square-headed stakes, accurately in the line of the base and at tape lengths from each other, starting from one end of the base. The tops of the stakes should be just flush with the ground; on these stakes nail strips of wood or zinc.

In measuring the base, stretch the tape from terminal point to first stake and from stake to stake; the tension should be taken on a spring balance and should be that given for the standardized tape. The end of the tape should be marked by a fine pencil line or knife cut on a wood or zinc strip nailed to top of each stake. The temperature of the tape should be taken at frequent intervals by letting the bulb of the thermometer come in contact with the underside of the tape.

At the other terminal the small space between the last graduation and the end of the base should be measured with a pair of dividers.

Now measure the inclination to horizontal from peg to peg with level or transit. Minor undulations crossed by the tape are to be disregarded.

The base should be measured **at least once in each direction**; on a dull day, if possible, or in early morning or late afternoon.

Base calculations.—The measured length of the base is subject to five corrections:

I. For standard; II. For temperature; III. For inclination; IV. For sag; V. For height above sea.

The following case may be taken as an example (taken from Close):

One tape was kept as a reference tape (the standardized one) and not used for measuring. This, when standardized, had been found to be 0.05 foot short at 62° F.

The tape used (supposed to be 100 feet long) was 0.02 foot long on **reference tape** at 71° F.

The temperature of tape used in the measurement was 82° F.

Height of base above sea, 4,520 feet.

For 2,300 feet, base had a slope of 1° 15′; remainder level.

I. Standard:

Reference tape was 0.05 short at 62° F.

Steel expands 0.00000625 of its length for 1° F.

Therefore, at 71° it was $9 \times 100 \times 62.5/10{,}000{,}000$ longer, or 0.0056 foot longer, or 0.05 less 0.0056 or 0.0444 short of standard.

Reference tape was therefore..	0.0444 short at 71° F.
Tape used....................	0.02 long on reference tape at 71° F.
Tape used was therefore.......	0.0244 short at 71° ...... (a).

II. Temperature:

Temperature of tape used during measurement was 32°; when compared was 71°.

Increase of length, 11×100×62.5/10,000,000, or 0.0069 ...... (*b*).

Combining (*a*) and (*b*) we find that the tape used was 0.0175 short, during measurement; correction for standard and temperature is therefore 0.0175×5301/100, or 0.928 foot.

III. Inclination:

For 2,300 feet the base has a slope of 1° 15′.

Correction, 2,300 (1–cos 1° 15′)=0.55.

IV. Sag:

In the measurement of a base it is desirable that the whole length of the tape should as far as possible be supported by the ground, so that no correction is necessary for the sag of the tape. Where the ground is uneven, it is customary to support the tape at intervals by stakes, but it may happen in the measurement of a base that a ravine has to be crossed. In such a case the sag, i. e., the difference between the length when suspended and when laid on a plane surface, must be determined and corrected for.

If s=the correction for sag,
l=the length of the tape suspended between two supports (in feet),
w=the weight of the tape in pounds,
t=the tension applied in pounds.
$s=l\, w^2/24t^2$.

V. Height above sea:

If the base is measured at a mean height (h) above sea level, it will require a correction of h/R×length of base, where R, the radius of the earth, may be taken as 20,900,000 feet.

In this case h=4,520. Then correction=4,520×5,301/20,900,000=1.146.

Collecting these corrections:	Feet.
I and II, standard and temperature	−0.928
III, inclination	−0.550
IV, sag	−0.000
V, height above sea	−1.146
Total correction	−2.624
Measured length	5,301.240
Corrected length of base	5,298.62

Corrections III and IV must always be negative; I and II may be either positive or negative.

189. For measurements requiring an accuracy greater than 50,000 the precaution taken is more elaborate in the following particulars. The tape is usually supported at intervals of 20 feet by adjustable standards, all standards being aligned and set to uniform grade for each tape length. The correction for sag may then be dispensed with.

For securing steady tension, several devices exist. The simplest is a pointed lever (e. g., a survey sighting rod). A movable collar with a hook or ring and secured by a set screw slides up and down the rod. The end of the tape is secured to this collar at any desired height above the ground. The toe of rod is stuck in the ground and the required tension secured by hauling back on the rod.

For greater accuracy still, the tape ends are secured to movable boxes which are weighted with stones. The ends of the graduation pass over wooden cages spiked into the ground. On the top of the cages are zinc plates for scratching the ends of tape lengths.

For detailed methods of the measurements of lines calling for high accuracy see treatises on surveying.

There are but few localities in the United States that can not conveniently be connected with known positions and distances and therefore, before base-line measurements are undertaken, the records of the Coast and Geodetic Survey, the Lake Survey, the Geological

Survey, the Corps of Engineers, United States Army, and other Government organizations should be examined in order to ascertain what positions in the area surveyed have been determined and are available for use in the work at hand.

MEASUREMENT OF DISTANCES BY STADIA.

190. The stadia is primarily intended to secure rapidity rather than accuracy; nevertheless with proper care to eliminate the chief sources of error, a high degree of accuracy may be obtained. Where properly handled, it will produce results as good as, and frequently better than, those with the tape, especially in rough country where variations in the slope of the ground affect taping seriously.

The degree of precision is dependent upon several conditions, chief among which are:

1. Length of sight; 2. Magnifying power of the telescope; 3. Fineness of the cross hairs; and 4. Precautions taken to avoid errors due to refraction.

Experiments show that the average error increases rapidly for sights over 800 feet. For short sights errors are less with a 25-power than with a 15-power telescope. On the Mexican boundary survey, the ratio of error between triangulation and taping was 1 in 1,436, and between triangulation and stadia 1 in 1,166. In all there were measured 182.5 miles by stadia which were triangulated and in which the total difference in length was 50 meters, or 1 in 5,873. Experiments show that refraction is a variable quantity, dependent upon temperature of air and ground, that is much greater near the ground than 3 feet above it; also at noon than before or after; that the effects vary for different distances. Twelve miles of stadia measurements with sights averaging 600 feet, in the morning and evening hours, showed an accuracy of 1 in 2,685. The same distances measured at midday showed an accuracy of 1 in 655.

191. **To obtain accuracy in stadia work** it is best to make short sights, avoid readings near the ground (bottom of the rod), and to avoid working during the midday hours, except on dull, overcast days. For accurate work the most approved practice is to use rods of standard division and to determine the rod interval factor to be applied to the observed distance as a correction. The speed of stadia traverses is far greater than that of taping where the surface of the ground is rough, since sights of one to two thousand feet length can be taken. Under similar circumstances the chain has to be laid down and stretched every hundred feet or less. Over smooth country, traverses may be made by stadia still more rapidly than by taping if the rodmen be mounted and the instrument man rides or drives. On the Mexican survey above cited as many as 16 miles a day were often covered, including the determination of height and the sketching of topography.

THEORY OF STADIA MEASUREMENTS.

192. The relation between the size and distance of an object and the size of its image in the telescope is given by the expression

$$Y^1/Y = F/X$$

where Y denotes the height of the object, Y^1 that of its image (the distance between stadia wires in this case), F the focal length of the

object glass, and X the distance of the object (rod) from the first principal focal point. This point lies in front of the object glass at a distance equal to the focal length (distance between the object glass and the stadia wires). To reduce the measured distance X to the true distance from the center of the instrument, add to X a constant equal to the distance of the first principal focal point from the center of the instrument. This constant is called c+f. Its value varies for different instruments and is usually marked on the inside of the transit box by the makers. It may be directly measured, however, with all the accuracy necessary. F is the distance from the object glass to the cross hairs when the object glass is focused for a distant object, and C is the distance from the object glass to the center of the instrument when focused for an average sight. The average value of c+f for a transit is about 1 foot.

GRADUATING THE STADIA ROD.

193. Rods should be of light, straight-grained, well-seasoned wood, 12 to 14 feet long, 5 inches wide, and $\frac{7}{8}$ inches thick, dressed smooth all around and covered with at least two coats of white paint. To graduate the rod it is necessary to know what space on the rod corresponds to a hundred feet in distance. Measure off c+f from the plumb bob and set a point. From this point measure off any convenient distance on level ground, as 500 feet. Hold the blank rod in a vertical position at the end of this base, or if possible fasten it rigidly in that position. Have a fixed mark or target on the upper part of the rod on which the upper wire is set. Have an assistant record the position of the lower wire as he is directed by the observer. Some sort of open target is good for this purpose, but any scheme is suitable that will enable the observer to fix the position of the extreme wires at the same moment with exactness. The work should be done when there is no wind and when the atmosphere is very steady; a calm day is best. Repeat the observation until the number of results, or their accordance, shows that the mean will give a good result. Divide the mean space so obtained into five equal parts, thus obtaining the intercept on the rod for 100 feet. By dividing this distance into 20 equal parts diagrams similar to Fig. 24 can be constructed and painted on the rod.

A method of graduation in common use is to divide all rods exactly in feet and tenths. This method has the advantages that the rod may be used as a level rod, that the same accurate template may be used to graduate all rods, etc. By the use of standard templates for patterning the rods large numbers of rods may be painted or repainted each season with much facility and with the minimum risk of mistakes.

194. **The stadia is used** in connection with a transit having a vertical limb, or a plane table with telescopic alidade having a vertical limb. Distances are read by setting either the upper or lower stadia hairs on a foot mark by means of the vertical arc clamp and tangent screw and counting the feet, tenths, and hundredths between the stadia hairs. The vertical angles are taken by sighting the middle cross hair on a point on the rod whose distance above the foot of the rod is equal to the distance from the horizontal axis of the telescope to the station beneath the transit. The distance is

known as the height of instrument (HI). It is convenient to mark this point on the rod by a rubber band which is adjusted for each set up.

The order of field work is as follows: First read the distance and record it, then set the middle hair on the rubber band and the vertical hair on the middle of the rod. Signal the rodman away and while he is moving to next point read the azimuth and the vertical angle.

REDUCTION OF STADIA FIELD NOTES.

195. This is done by means of tables, diagrams, or stadia slide rule rather than by direct use of formulæ. Table III gives, for vertical angles up to 20°, the difference in elevation for an inclined reading of 100 feet. If the inclined reading is 612, the instrumental constant (F+c) 1 foot, and the vertical angle 4° 42′, the difference in elevation is $6.13 \times 8.17 = 50.1$ feet. The horizontal distance is obtained by use of the table of **horizontal corrections** (Table II), which gives the distances to be subtracted from the inclined reading increased by the instrumental constant, c+f.

Stadia reduction tables are published (such as the Noble and Casgrain) in which the corrections are made by addition only, instead of multiplication as in Table III. These tables are more convenient than multiplication tables.

The most rapid method of reducing stadia readings is by means of stadia slide rules or computers, such as the Kern, the Colby, the Webb, the Matthes, the Cox, etc. For accurate traverses the readings should be reduced by table and checked by slide rule. For less accurate work the slide rule alone is sufficient.

PLOTTING.

196. The plotting of stadia notes which are "cold" is fraught with error. They should when practicable be plotted in the field **in the sight of the facts.** This may be done by means of a circular protractor and scale, a small plane table or sketching board being used as a plotting board. Some surveyors prefer to use the small plane table **as a plane table** in conjunction with the transit; the plane table being used to obtain and plot all directions, the transit being used as an auxiliary instrument to read distances and vertical angles.

CAUTION.

It is sometimes desirable to take long sights where the intercept of the stadia wires exceeds the length of the rod. In such cases it should be remembered that the middle wire is seldom exactly midway between the upper and lower wires. The sum of the readings between the upper and middle wire and between the middle and lower wire will give closer results than double the reading between the middle and one extreme wire. For accurate azimuth reading, bisect with the vertical wire the edge of the rod instead of the face.

TABLE III.—*Reductions of stadia observations.*

VERTICAL HEIGHTS.

	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	12°	13°	14°	15°	16°	17°	18°	19°
Minutes:																				
0	0.00	1.74	3.49	5.23	6.96	8.68	10.40	12.10	13.78	15.45	17.10	18.73	20.34	21.92	23.47	25.00	26.50	27.96	29.39	30.78
2	0.06	1.80	3.55	5.28	7.02	8 74	10.45	12.15	13.84	15.51	17.16	18.78	20.39	21.97	23.52	25.05	26.55	28.01	29.44	30.83
4	0.12	1.86	3.60	5.34	7.07	8.80	10.51	12.21	13.89	15.56	17.21	18.84	20.44	22.02	23.58	25.10	26.59	28.06	29.48	30.87
6	0.17	1.92	3.66	5.40	7.13	8.85	10.57	12.26	13.95	15.62	17.26	18.89	20.50	22.08	23.63	25.15	26.64	28.10	29.53	30.92
8	0.23	1.98	3.72	5.46	7.19	8.91	10.62	12.32	14.01	15.67	17.32	18.95	20.55	22.13	23.68	25.20	26.69	28.15	29.58	30.97
10	0.29	2.04	3.78	5.52	7.25	8.97	10.68	12.38	14.06	15.73	17.37	19.00	20.60	22.18	23.73	25.25	26.74	28.20	29.62	31.01
12	0.35	2.09	3.84	5.57	7.30	9.03	10.74	12.43	14.12	15.78	17.43	19.05	20.66	22.23	23.78	25.30	26.79	28.25	29.67	31.06
14	0.41	2.15	3.90	5.63	7.36	9.08	10.79	12.49	14.17	15.84	17.48	19.11	20.71	22.28	23.83	25.35	26.84	28.30	29.72	31.10
16	0.47	2.21	3.95	5.69	7.42	9.14	10.85	12.55	14.23	15.89	17.54	19.16	20.76	22.34	23.88	25.40	26.89	28.34	29.76	31.15
18	0.52	2.27	4.01	5.75	7.48	9.20	10.91	12.60	14.28	15.95	17.59	19.21	20.81	22.39	23.93	25.45	26.94	28.39	29.81	31.19
20	0.58	2.33	4.07	5.80	7.53	9.25	10.96	12.66	14.34	16.00	17.65	19.27	20.87	22.44	23.99	25.50	26.99	28.44	29.86	31.24
22	0.64	2.38	4.13	5.86	7.59	9.31	11.02	12.72	14.40	16.06	17.70	19.32	20 92	22.49	24.04	25.55	27.04	28.49	29.90	31.28
24	0.70	2.44	4.18	5.92	7.65	9.37	11.08	12.77	14.45	16.11	17.76	19.38	20.97	22.54	24.09	25.60	27.09	28.54	29.95	31.33
26	0.76	2.50	4.24	5.98	7.71	9.43	11.13	12.83	14.51	16.17	17.81	19.43	21.03	22.60	24.14	25.65	27.13	28.58	30.00	31.38
28	0.81	2.56	4.30	6.04	7.76	9.48	11.19	12.88	14.56	16.22	17.86	19.48	21.08	22.65	24.19	25.70	27.18	28.63	30.04	31.42
30	0.87	2.62	4.36	6.09	7.82	9.54	11.25	12.94	14.62	16.28	17.92	19.54	21.13	22.70	24.24	25.75	27.23	28.68	30.09	31.47
32	0.93	2.67	4.42	6.15	7.88	9.60	11.30	13.00	14.67	16.33	17.97	19.59	21.18	22.75	24.29	25.80	27.28	28.73	30.14	31.51
34	0.99	2.73	4.48	6.21	7.94	9.65	11.36	13.05	14.73	16.39	18.03	19.64	21.24	22.80	24.34	25.85	27.33	28.77	30.19	31.56
36	1.05	2.79	4.53	6.27	7.99	9.71	11.42	13.11	14.79	16.44	18.08	19.70	21.29	22.85	24.39	25.90	27.38	28.82	30.23	31.60
38	1.11	2.85	4.59	6.33	8.05	9.77	11.47	13.17	14.84	16.50	18.14	19.75	21.34	22.91	24.44	25.95	27.43	28.87	30.28	31.65
40	1.16	2.91	4.65	6.38	8.11	9.83	11.53	13.22	14.90	16.55	18.19	19.80	21.39	22.96	24.49	26.00	27.48	28.92	30.32	31.69
42	1.22	2.97	4.71	6.44	8.17	9.88	11.59	13.28	14.95	16.61	18.24	19.86	21.45	23.01	24.55	26.05	27.52	28.96	30.37	31.74
44	1.28	3.02	4.76	6.50	8.22	9.94	11.64	13.33	15.01	16.66	18.30	19.91	21.50	23.06	24.60	26.10	27.57	29.01	30.41	31.78
46	1.34	3.08	4.82	6.56	8.28	10.00	11.70	13.39	15.06	16.72	18.35	19.96	21.55	23.11	24.65	26.15	27.62	29.06	30.46	31.83
48	1.40	3.14	4.88	6.61	8.34	10.05	11.76	13.45	15.12	16.77	18.41	20.02	21.60	23.16	24.70	26.20	27.67	29.11	30.51	31.87
50	1.45	3.20	4.94	6.67	8.40	10.11	11.81	13.50	15.17	16.83	18.46	20.07	21.66	23.22	24.75	26.25	27.72	29.15	30.55	31.92
52	1.51	3.26	4.99	6.73	8.45	10.17	11.87	13.56	15.23	16.88	18.51	20.12	21.71	23.27	24.80	26.30	27.77	29.20	30.60	31.96
54	1.57	3.31	5.05	6.79	8.51	10.22	11 93	13.61	15.28	16.94	18.57	20.18	21.76	23.32	24.85	26.35	27.81	29.25	30.65	32.01
56	1.63	3.37	5.11	6.84	8.57	10.28	11.98	13.67	15.34	16.99	18.62	20.23	21.81	23.37	24.90	26.40	27.86	29.30	30.69	32.05
58	1.69	3.43	5.17	6.90	8.63	10.34	12.04	13.73	15.40	17.05	18.68	20.28	21.87	23.42	24.95	26.45	27.91	29.34	30.74	32.09
60	1.74	3.49	5.23	6.96	8.68	10.40	12.10	13.78	15.45	17.10	18.73	20.34	21.92	23.47	25.00	26.50	27.96	29.39	30.78	32.14

HORIZONTAL CORRECTIONS.

Distance:																				
100	0.0	0.0	0.1	0.3	0.5	0.8	1.1	1.5	1.9	2.5	3.0	3.6	4.3	5.1	5.9	6.7	7.6	8.5	9.5	10.6
200	0.0	0.1	0.2	0.5	1.0	1.5	2.2	3.0	3.9	4.9	6.0	7.3	8.6	10.1	11.7	13.4	15.2	17.1	19.1	21.2
300	0.0	0.1	0.4	0.8	1.5	2.3	3.3	4.5	5.8	7.4	9.1	10.9	13.0	15.2	17.6	20.1	22.8	25.6	28.6	31.8
400	0.0	0.1	0.5	1.1	2.0	3.0	4.4	6.0	7.8	9.8	12.1	14.6	17.3	20.2	23.4	26.8	30.4	34.2	38.2	52.4
500	0.0	0.2	0.6	1.4	2.5	3.8	5.5	7.5	9.7	12.3	15.1	18.2	21.6	25.3	29.3	33.5	38.0	42.7	47.7	43.0
600	0.0	0.2	0.7	1.6	2.9	4.6	6.5	8.9	11.6	14.7	18.1	21.8	25.9	30.4	35.1	40.2	45.6	51.3	57.3	63.6
700	0.0	0.2	0.8	1.9	3.4	5.3	7.6	10.4	13.6	17.2	21.1	25.5	30.2	35.4	41.0	46.9	53.2	59.8	66.8	74.2
800	0.0	0.2	1.0	2.2	3.9	6.1	8.7	11.9	15.5	19.6	24.2	29.1	34.6	40.5	46.8	53.6	60.8	68.4	76.4	84 8
900	0.0	0.3	1.1	2.4	4.4	6.8	9.8	13.4	17.5	22.1	27.2	32.8	38.9	45.5	52.7	60.3	68.4	76.9	85.9	95.4
1,000	0.0	0.3	1.2	2.7	4.9	7.6	10.9	14.9	19.4	24.5	30.2	36 4	43.2	50.6	58.5	67.0	76.0	85.5	95.5	106.0

DETERMINATION OF DIFFERENCES IN ELEVATION.

197. In topographical work elevations are referred to a common level surface called the **datum.** The datum is taken low enough so that no point of the area to be mapped will be below it. This makes all elevations positive. For topographic surveys the datum in general use throughout the world is mean sea level. This should always be used where practicable.

The difference in elevation of the two points may be determined: (*a*) By means of the angle which the line connecting the two points makes with the horizontal, and the horizontal or inclined distance between the two points; (*b*) by means of an aneroid barometer carried from one point to the other; (*c*) by means of the differential spirit leveling between the two points.

The vertical angle may be read with some form of clinometer, or with a transit having a vertical circle.

BAROMETRIC LEVELING.

198. The weight of the atmosphere at sea level is 14.703 pounds per square inch, equal to the weight of a column of mercury 29.92 inches high, or a column of fresh water 34.7 feet high.

The **aneroid barometer** records the pressure of the atmosphere in inches, the same as a mercurial barometer, the reading being taken from a pointer moving on a circular scale. It must be carefully handled as it is sensitive to shocks. A screw head will be seen through a hole in the back of the outer case by which the needle may be brought to any desired reading, and the instrument corrected whenever it can be compared with a standard. With the aneroid, corrections for instrumental temperature can not be made, and for this reason small pocket instruments are preferable, as carried in the pocket they are not exposed to so great changes in this respect.

The **pressure of the atmosphere varies** with the altitude above sea level, and it also varies with the moisture, temperature, and latitude, which do not depend upon the altitude.

In measuring altitudes with the barometer these other causes of variation must be eliminated so far as possible. It is best done by simultaneous observation at both stations. If the stations are not far apart all disturbing conditions will be substantially the same at each and therefore eliminated, except temperature, which, with considerable difference of altitude, will always be less at the upper than at the lower station.

If *simultaneous* observations *can not* be made, the stations should be occupied with as *little interval of time* between as possible, and better results will be obtained if the time of observation can be so chosen as to take advantage of calm, bright, dry weather.

When the hygrometric conditions are very uniform an aneroid read at intervals on a day's march over a rough conutry will give a fairly good idea of the profile.

199. **Table of elevations** above sea level from barometer readings (United States Coast and Geodetic Survey), for mean hygrometric conditions and mean temperature of 50° F.:

Barometer reading.	Altitude above sea level.	Differential for 0.01 inch.	Barometer reading.	Altitude above sea level.	Differential for 0.01 inch.	Barometer reading.	Altitude above sea level.	Differential for 0.01 inch.
Inches.	*Feet.*	*Feet.*	*Inches.*	*Feet.*	*Feet.*	*Inches.*	*Feet.*	*Feet.*
18.0	13,918	−15.1	22.2	8,204	−12.2	26.4	3,483	−10.3
18.1	13,767	−15.0	22.3	8,082	−12.2	26.5	3,380	−10.3
18.2	13,617	−14.9	22.4	7,960	−12.2	26.6	3,277	−10.2
18.3	13,468	−14.9	22.5	7,838	−12.1	26.7	3,175	−10.2
18.4	13,319	−14.7	22.6	7,717	−12.0	26.8	3,073	−10.1
18.5	13,172	−14.7	22.7	7,597	−12.0	26.9	2,972	−10.1
18.6	13,025	−14.6	22.8	7,477	−11.9	27.0	2,871	−10.1
18.7	12,879	−14.6	22.9	7,358	−11.9	27.1	2,770	−10.0
18.8	12,733	−14.4	23.0	7,239	−11.8	27.2	2,670	−10.0
18.9	12,589	−14.4	23.1	7,121	−11.7	27.3	2,570	−10.0
19.0	12,445	−14.3	23.2	7,004	−11.7	27.4	2,470	− 9.9
19.1	12,302	−14.2	23.3	6,887	−11.7	27.5	2,371	− 9.9
19.2	12,160	−14.2	23.4	6,770	−11.6	27.6	2,272	− 9.9
19.3	12,018	−14.1	23.5	6,654	−11.6	27.7	2,173	− 9.9
19.4	11,877	−14.0	23.6	6,538	−11.5	27.8	2,075	− 9.8
19.5	11,737	−13.9	23.7	6,423	−11.5	27.9	1,977	− 9.8
19.6	11,598	−13.9	23.8	6,308	−11.4	28.0	1,880	− 9.7
19.7	11,459	−13.8	23.9	6,194	−11.4	28.1	1,783	− 9.7
19.8	11,321	−13.7	24.0	6,080	−11.3	28.2	1,686	− 9.7
19.9	11,184	−13.7	24.1	5,967	−11.3	28.3	1,589	− 9.7
20.0	11,047	−13.6	24.2	5,854	−11.3	28.4	1,493	− 9.6
20.1	10,911	−13.5	24.3	5,741	−11.2	28.5	1,397	− 9.6
20.2	10,776	−13.4	24.4	5,629	−11.1	28.6	1,302	− 9.5
20.3	10,642	−13.4	24.5	5,518	−11.1	28.7	1,207	− 9.5
20.4	10,508	−13.3	24.6	5,407	−11.1	28.8	1,112	− 9.5
20.5	10,375	−13.3	24.7	5,296	−11.0	28.9	1,018	− 9.4
20.6	10,242	−13.2	24.8	5,186	−10.9	29.0	924	− 9.4
20.7	10,110	−13.1	24.9	5,077	−10.9	29.1	830	− 9.4
20.8	9,979	−13.1	25.0	4,968	−10.9	29.2	736	− 9.3
20.9	9,848	−13.0	25.1	4,859	−10.8	29.3	643	− 9.3
21.0	9,718	−12.9	25.2	4,751	−10.8	29.4	550	− 9.2
21.1	9,589	−12.9	25.3	4,643	−10.8	29.5	458	− 9.2
21.2	9,460	−12.8	25.4	4,535	−10.7	29.6	366	− 9.2
21.3	9,332	−12.8	25.5	4,428	−10.7	29.7	274	− 9.2
21.4	9,204	−12.7	25.6	4,321	−10.6	29.8	182	− 9.1
21.5	9,077	−12.6	25.7	4,215	−10.6	29.9	91	− 9.1
21.6	8,951	−12.6	25.8	4,109	−10.5	30.0	00	− 9.1
21.7	8,825	−12.5	25.9	4,004	−10.5	30.1	− 91	− 9.0
21.8	8,700	−12.5	26.0	3,899	−10.5	30.2	−181	− 9.0
21.9	8,575	−12.4	26.1	3,794	−10.4	30.3	−271	− 9.0
22.0	8,451	−12.4	26.2	3,690	−10.4	30.4	−361	− 9.0
22.1	8,327	−12.3	26.3	3,586	−10.3	30.5	−451	− 8.9

COEFFICIENTS FOR TEMPERATURE CORRECTION.

200. Argument $(t+t')$ = Sum of temperatures at the two stations:

$t+t'$.	Coefficient C.	$t+t'$.	Coefficient C.	$t+t'$.	Coefficient C.
°		°		°	
0	−0.1024	70	−0.0273	130	+0.0368
10	−0.0915	80	−0.0166	140	+0.0472
20	−0.0806	90	−0.0058	150	+0.0575
30	−0.0698	100	+0.0049	160	+0.0677
40	−0.0592	110	+0.0156	170	+0.0779
50	−0.0486	120	+0.0262	180	+0.0879
60	−0.0380				

Examples:

Station.	Barometer.	Temperature.
	Inches.	°*F.*
Sacramento	30.014	59.9
Summit	23.288	42.1

From table of elevations:

	Feet.
Sacramento	= −12.7
Summit	=6,901.0
Differential	=6,913.7
$t+t'$	=102°
∴ C	= 0.0070
∴ Temperature correction, 6,913.7×0.007	= +48.4
H	=6,962.1

Station.	Barometer.	Temperature.
	Inches.	°*F.*
Lower	28.075	57.3
Upper	22.476	38.5

From table of elevations:

	Feet.
Lower	= 7,867.0
Upper	= 1,807.0
Differential	= 6,060.0
$t+t'$	= 95°.08
∴ C	= +0.0004
∴ Temperature correction, 6,060 ×0.0004	= +2.4
H	= 6,062.4

GENERAL RULES FOR USING ANEROID BAROMETERS.

201. The best type of aneroid barometer for use in reconnaissance is one with a dial about 2¾ inches in diameter, graduated to 3,000 feet on the scale, with a least reading of 10 feet. In using the barometer—

(1) Keep it at a temperature as nearly constant as is practicable. This is best done by keeping it in an inner pocket, where it will have nearly the temperature of the body. Remove it from the pocket only for the purpose of reading and return it as soon as possible.

(2) Always hold the barometer with its dial horizontal when reading it and tap it gently two or three times with the finger or pencil before reading.

(3) In clear settled weather it will be found that the pressure variation due to change of temperature follows a regular law. Beginning at about 9 a. m. the elevation scale will show a rise of about 10 feet per hour for about four hours. It will then remain stationary until about 4 p. m., and will then fall regularly until about 7 p. m., when the same reading as at 9 a. m. will be reached. A knowledge of this change will enable proper corrections to be made.

(4) In unsettled weather, before or after a storm, note, if possible, the movement of the needle for an hour before starting work to ascertain its direction and rate of change, and thus be enabled to make proper corrections.

LEVELING WITH THE HAND LEVEL.

202. Differences in elevation can be determined with considerable accuracy by means of the hand level or by means of the clinometer, using level sights. For reconnaissance work without an assistant,

leveling is done up grade; note where the level line of sight strikes the ground, advance to that point and repeat the operation. Each advance corresponds to a difference of elevation equal to the height of the observer's eye. If an assistant is available, leveling can be done down grade as well as up, in which case much longer sights are possible by the use of an improvised level rod.

The hand level in connection with a standard level rod, carried by an assistant, has a wide application in construction work. It admits of great rapidity in cross-section leveling and gives results sufficiently accurate for construction purposes.

The locator's hand level, combining the virtues of the hand level and the clinometer, is an instrument of peculiar value in all reconnaissance work.

THE ENGINEER'S LEVEL.

203. This instrument is shown and its parts indicated in Fig. 29. The instrument is focused and set up as described for the transit, except that, as there is but one level, the telescope must be turned in the direction of one pair of leveling screws and leveled, then turned in the direction of the other pair and leveled again. The second leveling usually disturbs the first and the latter should then be releveled.

The level consists essentially of two geometric straight lines—the line of sight and the vertical axis. The adjustment of the instrument consists in making these two lines truly perpendicular to each other.

This is effected by the use of the level tube. **The line of sight** (the geometric line through the center of the object glass and the intersection of the cross hairs) is made parallel to the **axis of the bubble tubes** by making each parallel to the axis of the Y's. The verticality of the vertical axis, in the adjusted instrument, is secured by the operation of leveling.

First, make the line of sight parallel to the axis of the Y's, as follows: Set up the instrument and level carefully; note a small object about 300 feet away that one end of the horizontal cross hair touches; turn the instrument in azimuth a few degrees and note whether the other end of the cross wire cuts the point; if it does the horizontal wire is horizontal. **Now** unlock the Y clips. Bisect with the intersection of the cross hairs some sharply defined point at a convenient distance; **revolve (not reverse)** the telescope in the Y's until the bubble comes on top. If there is not coincidence between the line **of sight and the axis of the Y's**, the intersection of the cross hairs will, in the revolved position, not be in the original mark and the amount that it has moved measures **double the error.** Correct the horizontal hair, moving it (by tightening the top and loosening the bottom capstan screws which hold the reticle or by tightening the bottom and loosening the top, as the case may be) **halfway back toward the original mark. Verify** the correction by repeating the entire operation. Two attempts should usually be sufficient to eliminate all error. **Then adjust the vertical wire in the same way.**

Second: Make the bubble axis parallel to the axis of the Y's, as follows: Set up the instrument and unlock the Y clips. Bring the telescope to a position over one diagonal set of leveling screws and clamp in azimuth. Using the leveling screws, now center the bubble. Then lift the telescope carefully out of the Y's and replace it in them,

reverse end for end. If the **bubble axis is not parallel to the axis of the Y's,** it will be shown by the bubble leaving the center and the amount of movement measures twice the error. **Bring the bubble halfway back by means of the adjusting screws at one end of the level tube.**

Now verify the adjustment by repeating the entire operation. Two attempts should usually be sufficient to eliminate all error.

The line of sight and the axis of the bubble tube now being each parallel to the axis of the Y's are parallel to each other, and the level is in adjustment. This is the only essential adjustment of the level, and if the bubble be centered carefully in every new direction that a sight is taken there will be no error in the work.

It is desirable, however, as a matter of convenience, that the vertical axis be made truly vertical, so that if the instrument is leveled in one direction the bubble will remain centered while the instrument is moved in azimuth.

To make the vertical axis truly vertical, proceed as follows:

The instrument must first be adjusted as above indicated. Set up the instrument and bring the telescope over a diagonal set of leveling screws. Center the bubble **by the leveling screws. Turn the instrument 180° in azimuth.** If the vertical axis is not truly vertical, the bubble will leave the center, and the amount of movement will measure twice the error. Correct by moving the bubble halfway back **by means of the large nut under one Y.** Verify the adjustment, and, if necessary, repeat.

"**Peg method.**" **The method of adjustment of the level given assumes that the two rings on the telescope tube which rest in the Y's are circular and exactly equal by construction. This is looked to by good instrument makers.**

The line of sight and axis of the bubble may be made perpendicular independently of these two rings and the axis of the Y's by the method known as the "peg method." This method is described in full under the adjustment of the transit, paragraph 166.

Level rods are of two kinds, target and **self-reading or speaking.** The target rod is finely graduated and has a metal target sliding on it, which is graduated as a vernier. The levelman signals to the rodman, who moves the target up or down until it is in the correct position, when the reading is taken by the rodman, or else the rod is carried to the levelman to be read. The ordinary form is the New York rod. The rod proper is in two parts, which slide on each other. For readings up to 6.5 feet the target is moved on the rod and read from the graduation on the front part by a vernier on the target. For greater readings the target is clamped at 6.5 feet and the back part of the rod slid up on the front part, the reading being taken from a scale on the *side* of the back part of the vernier on the *side* of the front part. The rod is graduated to hundredths of feet and the verniers read to thousandths.

FIELD NOTES FOR PROFILE LEVELING.

204.

Station.	B. S.	H. I.	F. S.	Elevation.	B. M. and T. P. elevation.	Remarks.
B. M. 16	7.825	115.089			107.264	x cut on east end of south abutment of Main Street Bridge over Jones Creek.
0			6.32	108.77		
1			5.01	110.08		
2			4.78	110.31		
3			3.22	111.87		
T. P.	7.326	119.978	2.437		112.652	
4			8.28	111.70		
4 70			7.32	112.66		
5			5.26	114.72		
6			3.15	116.83		
6 95			2.14	117.84		
7			2.05	117.93		
T. P.	9.326	127.867	1.437		118.541	x cut on northwest corner top step St. Lukes Church.
9			7.25	120.62		
	15.151 3.874		3.874			
	11.277 107,264					
	118.541					

FIELD NOTES FOR DIFFERENTIAL LEVELING.

Station.	B. S.	H. I.	F. S.	Elevation.	Remarks.
B. M. 21	8.752	383.008		374.256	x cut on northeast corner of coping of bridge corner Main and Second Streets.
T. P. 1	9.365	389.936	2.437	380.571	
T. P. 2	10.213	398.003	2.146	387.790	
T. P. 3	6.428	401.215	3.216	394.787	
B, M.			0.983	400.232	Station No. 3.
	34.758 8,782		8,782		
	25.976 374.256				
	400.232				

PROFILE LEVELING.

205. To determine a profile: The line to be profiled is first stationed, every 100 foot point or such other distance as is desired being distinctly marked, usually with a stake. The level is set up and a rod reading called a **back sight** (B. S.) taken on a point, called a **bench mark** (B. M.) whose elevation is known. When the B. S. is added to the elevation of the B. M., it gives the **height of the instrument** (H. I.). Rod readings called **fore sights** (F. S.), may then be read on as many station points as can be conveniently seen from the instrument. The elevation of the point on which the rod rests, when A. F. S. is taken is found by subtracting the F. S. from the H. I.

F. S.'s are taken to all station stakes and also to all changes of slope whether they fall at a station or not; and intermediate stations at such points are located by tape measurements. When all the required F. S.'s have been taken that are desired from first set up and it becomes necessary to move the instrument to a new

position to proceed along the line to be profiled, a **turning point** (T. P.) is selected and its elevation is determined by a careful F. S. **This elevation is to be used to determine the height of instrument at the new position.** This is done by setting up the instrument at the

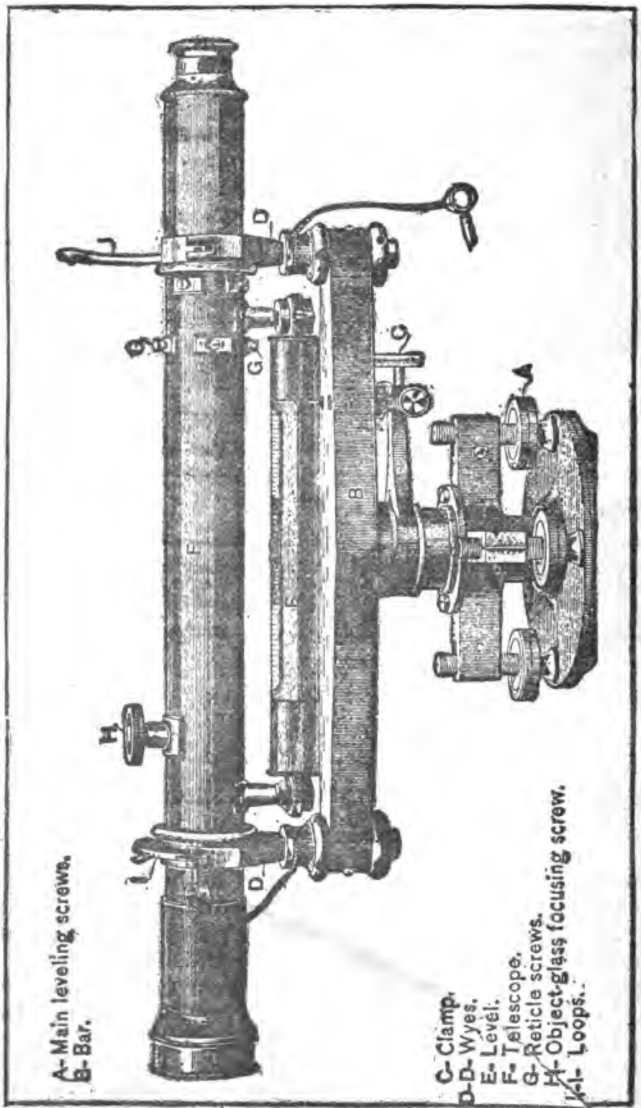


Fig. 29

new position and taking a back sight at the T. P. This B. S. added to the predetermined elevation of the T. P., is the H. I. for the new position. As soon as this is found F. S.'s may be taken on the line. Then a new T. P. is selected from which to determine the H. I. of a

third position of the level, etc. **Readings on B. M.'s and T. P.'s should be taken to one more decimal place than those for the profile.** The B. M.'s are all carefully described in the notes. As a rule the T. P.'s are not so described, as they are of temporary use only, except when they are taken on easily identified points.

In short lines where great accuracy is not needed, the T. P.'s may be stones or stakes driven into the ground at convenient points. When accurate work is attempted a better T. P. must be used; such as the head of a hatchet the blade of which is firmly driven into the ground, or a large stone well embedded; **still better**, a steel pin 8 inches long, carried by the rodman for the purpose. Such a pin should be driven into the ground for use and its top should be kept rounded by frequent dressing so that there shall be but one point of contact with the rod; as soon as the new H. I. has been determined from the T. P., the steel pin is pulled from the ground by a cord through an eye in the pin, and used at the next T. P.

The H. I. must always be obtained from a B. M. or proper T. P.; it should never be determined from a stake on the line which is not perfectly firm in the ground and provided with an upper surface giving only one point of contact.

DIFFERENTIAL LEVELING.

206. In differential leveling, properly speaking, the only result sought is the determination of the difference of elevation of two points, such as between an existing B. M. and one newly established, etc. No stationing of such a line is necessary and all sights taken are to T. P.'s or B. M.'s. Differential and profile leveling merge into each other and there are few cases in practice where the line run is not a combination of the two.

LEVEL NOTES.

207. The forms given in paragraph 207 are recommended. The column arrangement gives the order of work. All level lines should be checked as shown on the forms. **The difference between the sum of the B. S.'s** and the **sum of the F. S.'s**, between any two elevations, added or subtracted, as the case may be, **to the first elevation will give the other elevation.** This check applies only to the sights which are included in the chain of the levels. No F. S. on other than T. P.'s or B. M.'s are part of the chain of levels. **This check is arithmetic only and therefore exact.** It is in no way a check on the leveling, but only on the accuracy of the notes.

A level line is a curved line everywhere perpendicular to the plumb line, while the line of sight of the instrument is a horizontal line tangent to the level line through the instrument. In order, therefore, to run out a line of levels (curve) by means of a straight line of sight, it is necessary that the F. S. and B. S. be equal in length. The varying densities of layers of air cause a ray of light to be bent in a vertical direction. The effect of this bending (refraction) can be eliminated by limiting the sights to 300 to 400 feet and making the F. S. and B. S. equal. The boiling of the air may make necessary even shorter sights than these.

208. **The accuracy** of level lines may be checked as follows: **Double rodded lines may be run.** In such lines, for every set up of the instru-

ment two (2) T. P.'s are read. They should be close together but should vary in elevation by at least a foot. This gives two independent determinations for each H. I. When these independent determinations differ by small amounts, the work is accepted and **notes of both determinations are taken.** If they show sudden marked differences, the work is investigated on the spot till the error or mistakes are discovered and corrected. By this method the leveler knows at all times the measure of the precision of his work before he leaves the field.

Another method is to run what are called **level loops.** The level line is carried forward a mile or so and then closed back to the initial point. If the circuit closes within the allowable limit of error, the work is accepted, the error distributed properly; the work then proceeds from the outer end of the loops as a B. M. and a second loop is run, etc. **The allowable error** in the precise leveling of the United States Coast and Geodetic Survey is 4 millimeters multiplied by the square root of the distance run in kilometers. For such work self-reading level rods are used and in the construction of the instrument and its manipulation. The observations are made in such a manner as to eliminate the recognized common errors in leveling, which are setting of level on soft ground; unequal expansion and contraction of different parts of the instrument due to temperature changes; irregular refraction of air near the ground; unequal length of back and fore sights; poor turning points; rod not held plumb; bubble not centered when reading is taken. These errors may be eliminated in levels run with the engineer's level and results of considerable accuracy obtained. The ordinary Y level has given results well within the accuracy of precise work. The allowable error of the United States Geological Survey **primary levels** is equal to the square root of the number of miles run multiplied by 0.04 of a foot (0.04 foot-miles). Work of such accuracy is done only on the **main level control lines** of a survey; between points thus accurately determined are run level lines of a lesser accuracy by the level of the transit and stadia or by the aneroid, these latter classes of lines being adjusted to the more accurate lines.

SPEED OF LEVELING.

209. The speed with which levels may be run varies greatly with the accuracy desired, the character of the country run over, the atmospheric conditions, the method of leveling employed, and the skill of the levelmen and rodmen. **Engineering levels** of considerable accuracy, such as the **primary levels of the United States Geological Survey** are run at speeds varying from 50 to 90 miles per month; the precise levels of the Coast and Geodetic Survey have been at the rate of 3 to 5 miles per day (precise levels are, of course, usually run over the best and most favorable grades). Flying levels, in which T. P.'s only are taken, may be run under the most favorable conditions at a rate of over 10 miles per day.

210. **The logarithm of a number** is the exponent of the power to which a certain other number, called the base, must be raised to produce the given number. The base of the system most used, called **common logarithms,** is 10.

In any system—

The log. of a product equals the sum of the logs. of the factors.

The log. of a quotient equals the log. of the dividend minus the log. of the divisor; or the log. of a common fraction equals the log. of the numerator minus the log. of the denominator.

The log. of 1 is 0; since the log. 1 = the log. $\frac{1}{1}$ = the log. 1 − log. 1 = 0.

The log. of a power of a number equals the log. of the number multiplied by the exponent of the power. The log. of a root of a number equals the log. of the number divided by the index of the root.

The first property above is utilized in the construction of the tables. Each log. is the sum of the logs. of two factors of which its number is composed, and the factors may be so chosen that the log. of one is a whole number, called the **characteristic,** and the log. of the other is a decimal fraction, called the **mantissa.** Any number may be resolved into two factors, one of which is the number itself with the decimal point after the first significant figure, and the other the figure 1, alone, or followed or preceded by one or more ciphers.

Thus:

$3760=3.76\times1000$	log. $=3.57518$
$376=3.76\times100$	log. $=2.57518$
$37.6=3.76\times10$	log. $=1.57518$
$3.76=3.76\times1$	log. $=0.57518$
$0.376=3.76\times0.1$	log. $=\bar{1}.57518$
$0.0376=3.76\times0.01$	log. $=\bar{2}.57518$
$0.00376=3.76\times0.001$	log. $=\bar{3}.57518$

The log. of the constant factor, 3.76 in the above example, is always a positive decimal fraction, and is called the **mantissa.** The log. of the variable factor in the third column above is a whole number and may be positive or negative. It is called the **characteristic.** The logs. of all numbers presenting the same combination of significant figures have the same mantissa regardless of the position of the decimal point. **Logarithmic tables contain mantissas only,** since the characteristics may be written by inspection and mental calculation. To this rule tables of logarithmic circular functions are an exception, as will be explained later. If the number is **whole or mixed,** the characteristic of its log. is positive, and **one less** than the number of places of figures in the integral part, or on the **left of the decimal point. If** the number is a decimal fraction, the characteristic of its log. **is negative,** and **one greater** than the number of ciphers **immediately following the decimal point.** See example preceding. **If the characteristic is positive,** the log. is a mixed number and may be treated as such in addition, subtraction, multiplication, and division.

If the characteristic is negative, the log. is not a true mixed number and special treatment is necessary. A negative characteristic may be considered as composed of two numbers, one negative and the other positive. The positive number, prefixed to the mantissa, forms a mixed number for arithmetical operations. The positive and negative parts may be simultaneously increased numerically by the same number without altering the value of the log.

Thus:

$$\bar{3}.4281=\bar{3}+0.4281$$
$$=\bar{4}+1.4281$$
$$=\bar{5}+2.4281,\text{ etc.}$$

For example, to multiply $\bar{4}.7265$ by 4.

$$\begin{array}{r} \bar{4}+0.7265 \\ 4 \\ \hline \overline{16}+2.9060=\overline{14}.9060, \end{array}$$ which is the required result.

To subtract $\bar{1}.8432$ from $\bar{3}.1329=\bar{4}+1.1329$.

$$\begin{array}{r} \bar{4}+1.1329 \\ \bar{1}+0.8432 \\ \hline \bar{3}+0.2897=\bar{3}.2897 \end{array}$$

To divide $\bar{2}.2368$ by 7. $\bar{2}.2368=\bar{7}+5.2368$.

$$\frac{\bar{7}+5.2368}{7}=\bar{1}+0.7481=\bar{1}.7481$$

In this case the number added to the minus characteristic should be just enough to make it *exactly divisible* by the divisor.

In the logs. of circular functions a characteristic is given in the tables which is larger by 10 than the true characteristic. These logs. may be used by the above rule by prefixing $\overline{10}$ to each. Thus the log. sine of 21 min. as given in the table $=7.78594$. The true log. is $\overline{10}+7.78594$, or $\bar{3}.78594$. Those who are familiar with the use of these logs. perform the operation on the $\overline{10}$ mentally. The inexperienced will do well to write them out in full.

EXPLANATION OF THE TABLE.

211. Table IV gives to five decimal places the common logs. of numbers from 0 to 999 directly, and by interpolation from 0 to 9,999. If the log. of a number larger than 10,000 is desired, factor it and take the sum of the logs. of the factors. Thus, log. 99,225 = log. of 75,000 plus the log. of 1.323 = 4.87506 + 0.12156 = 4.99662. Or convert the number into a mixed number less than 1,000 and find its log. Thus, log. 992.25 = 992 + ¼ difference between 992 and 993 = 99,662, which is the mantissa for 99,225.

In the table the logs. of 2 to 9, inclusive, are found at the tops of the columns. For numbers above 10, the first two figures are in the first column, the third at the tops of the columns, and the fourth is interpolated. The right-hand column contains the average difference in each line between logs. in successive columns. For the fourth place multiply one-tenth of the difference on the same line by the fourth figure, and add the product to the log. of the first three figures. Thus:

To find the log. of 4,827, look for 48 in the left-hand column; follow the line to the column headed 2, and take out the mantissa 0.68304 for the number 482. In the right-hand column on the same line is the difference 90, one-tenth of which, 9, multiplied by the fourth figure, 7, = 63, to be added to the log. of 482, making the mantissa of 4,827 = 0.68367. The characteristic is 3 or 1 less than the number of places of integral figures in the number, hence the complete log. of 4,827 is 3.68367.

When the difference exceeds 200, if close results are desired, use the difference obtained by subtracting the number found for the third figure from that in the column for the next higher figure.

The number corresponding to any log. may be obtained from the table by the inverse process. If the given log. is found in the table, the corresponding number consists of the two figures on the left of

the line, followed by the one at the top of the column. If the exact log. is not in the table, find the next one below and take out the three figures for that. Take the difference between the given log. and the one found in the table next below it and divide this difference by one-tenth the tabulated difference on the line. Write down the quotient for the fourth figure of the required number.

Thus, to find the number corresponding to $\bar{1}.49638$. This is not in the table and the next below is 49,554. The two figures on the left of the line are 31 and the figure at top of column is 3. Hence 313 is the number corresponding to 49,554. The difference between 49,638 and 49,554 is 84, which divided by 14 or one-tenth of the tabulated difference 138 on the right of the line gives a quotient of 6 to be set down as the fourth figure. Hence the number required is 0.3136, since the characteristic is $\bar{1}$ and therefore the significant figures are immediately after the decimal point.

TABLE IV.

Common logarithms, 1 to 999:

No.	0	1	2	3	4	5	6	7	8	9	Diff.
		00000	30103	47712	60206	69897	77815	84510	90309	95424	
10	00000	00432	00860	01283	01703	02118	02530	02938	03342	03742	415
11	04139	04532	04921	05307	05690	06069	06445	06818	07188	07554	379
12	07918	08278	08636	08990	09342	09691	10037	10380	10721	11059	349
13	11394	11727	12057	12385	12710	13033	13353	13672	13987	14301	323
14	14613	14921	15228	15533	15836	16136	16435	16731	17026	17318	300
15	17609	17897	18184	18469	18752	19033	19312	19590	19865	20139	281
16	20412	20682	20951	21218	21484	21748	22010	22271	22530	22788	264
17	23045	23299	23552	23804	24054	24303	24551	24797	25042	25285	249
18	25527	25767	26007	26245	26481	26717	26951	27184	27415	27646	236
19	27875	28103	28330	28555	28780	29003	29225	29446	29666	29885	223
20	30103	30319	30535	30749	30963	31175	31386	31597	31806	32014	212
21	32222	32428	32633	32838	33041	33243	33445	33646	33845	34044	202
22	34242	34439	34635	34830	35024	35218	35410	35602	35793	35983	194
23	36173	36361	36548	36735	36921	37106	37291	37474	37657	37839	185
24	38021	38201	38381	38560	38739	38916	39093	39269	39445	39619	177
25	39794	39967	40140	40312	40483	40654	40824	40993	41162	41330	171
26	41497	41664	41830	41995	42160	42324	42488	42651	42813	42975	164
27	43136	43296	43456	43616	43775	43933	44090	44248	44404	44560	158
28	44716	44870	45024	45178	45331	45484	45636	45788	45939	46089	153
29	46240	46389	46538	46686	46834	46982	47129	47275	47421	47567	148
30	47712	47856	48000	48144	48287	48430	48572	48713	48855	48995	143
31	49136	49276	49415	49554	49693	49831	49968	50105	50242	50379	138
32	50515	50650	50785	50920	51054	51188	51321	51454	51587	51719	134
33	51851	51982	52113	52244	52374	52504	52633	52763	52891	53020	130
34	53148	53275	53402	53529	53655	53781	53907	54033	54157	54282	126
35	54407	54530	54654	54777	54900	55022	55145	55266	55388	55509	122
36	55630	55750	55870	55990	56110	56229	56348	56466	56584	56702	119
37	56820	56937	57054	57170	57287	57403	57518	57634	57749	57863	116
38	57978	58092	58206	58319	58433	58546	58658	58771	58883	58995	113
39	59106	59217	59328	59439	59549	59659	59769	59879	59988	60097	110
40	60206	60314	60422	60530	60638	60745	60852	60959	61066	61172	107
41	61278	61384	61489	61595	61700	61804	61909	62013	62118	62221	104
42	62325	62428	62531	62634	62736	62838	62941	63042	63144	63245	102
43	63347	63447	63548	63648	63749	63848	63948	64048	64147	64246	99
44	64345	64443	64542	64640	64738	64836	64933	65030	65127	65224	98
45	65321	65417	65513	65609	65705	65801	65896	65991	66086	66181	96
46	66276	66370	66464	66558	66651	66745	66838	66931	67024	67117	94
47	67210	67302	67394	67486	67577	67669	67760	67851	67942	68033	92
48	68124	68214	68304	68394	68484	68574	68663	68752	68842	68930	90
49	69020	69108	69196	69284	69372	69460	69548	69635	69722	69810	88
50	69897	69983	70070	70156	70243	70329	70415	70500	70586	70671	86
51	70757	70842	70927	71011	71096	71180	71265	71349	71433	71516	84
52	71600	71683	71767	71850	71933	72015	72098	72181	72263	72345	82
53	72428	72509	72591	72672	72754	72835	72916	72997	73078	73158	81
54	73239	73319	73399	73480	73559	73639	73719	73798	73878	73957	80
55	74036	74115	74193	74272	74351	74429	74507	74585	74663	74741	78
56	74818	74896	74973	75050	75127	75204	75281	75358	75434	75511	77
57	75587	75663	75739	75815	75891	75966	76042	76117	76192	76267	75
58	76342	76417	76492	76566	76641	76715	76789	76863	76937	77011	74
59	77085	77158	77232	77305	77378	77451	77524	77597	77670	77742	73
60	77815	77887	77959	78031	78103	78175	78247	78318	78390	78461	72

Table IV—Continued.

Common logarithms, 1 to 999—Continued.

No.	0	1	2	3	4	5	6	7	8	9	Diff.
61	78533	78604	78675	78746	78816	78887	78958	79028	79098	79169	71
62	79239	79309	79379	79448	79518	79588	79657	79726	79796	79865	70
63	79934	80002	80071	80140	80208	80277	80345	80413	80482	80550	69
64	80618	80685	80753	80821	80888	80956	81023	81090	81157	81224	68
65	81291	81358	81424	81491	81557	81624	81690	81756	81822	81888	67
66	81954	82020	82085	82151	82216	82282	82347	82412	82477	82542	66
67	82607	82672	82736	82801	82866	82930	82994	83058	83123	83187	65
68	83250	83314	83378	83442	83505	83569	83632	83695	83758	83821	64
69	83884	83947	84010	84073	84136	84198	84260	84323	84385	84447	63
70	84509	84571	84633	84695	84757	84818	84880	84941	85003	85064	62
71	85125	85187	85248	85309	85369	85430	85491	85551	85612	85672	61
72	85733	85793	85853	85913	85973	86033	86093	86153	86213	86272	60
73	86332	86391	86451	86510	86569	86628	86687	86746	86805	86864	59
74	86923	86981	87040	87098	87157	87215	87273	87332	87390	87448	58
75	87506	87564	87621	87679	87737	87794	87852	87909	87966	88024	57
76	88081	88138	88195	88252	88309	88366	88422	88479	88536	88592	56
77	88649	88705	88761	88818	88874	88930	88986	89042	89098	89153	56
78	89209	89265	89320	89376	89431	89487	89542	89597	89652	89707	55
79	89762	89817	89872	89927	89982	90036	90091	90145	90200	90254	54
80	90309	90363	90417	90471	90525	90579	90633	90687	90741	90794	54
81	90848	90902	90955	91009	91062	91115	91169	91222	91275	91328	53
82	91381	91434	91487	91540	91592	91645	91698	91750	91803	91855	53
83	91907	91960	92012	92064	92116	92168	92220	92272	92324	92376	52
84	92427	92479	92531	92582	92634	92685	92737	92788	92839	92890	51
85	92941	92993	93044	93095	93146	93196	93247	93298	93348	93399	51
86	93449	93500	93550	93601	93651	93701	93751	93802	93852	93902	50
87	93951	94001	94051	94101	94151	94200	94250	94300	94349	94398	49
88	94448	94497	94546	94596	94645	94694	94743	94792	94841	94890	49
89	94939	94987	95036	95085	95133	95182	95230	95279	95327	95376	48
90	95424	95472	95520	95568	95616	95664	95712	95760	95808	95856	48
91	95904	95951	95999	96047	96094	96142	96189	96236	96284	96331	48
92	96378	96426	96473	96520	96567	96614	96661	96708	96754	96801	47
93	96848	96895	96941	96988	97034	97081	97127	97174	97220	97266	47
94	97312	97359	97405	97451	97497	97543	97589	97635	97680	97726	46
95	97772	97818	97863	97909	97954	98000	98045	98091	98136	98181	46
96	98227	98272	98317	98362	98407	98452	98497	98542	98587	98632	45
97	98677	98721	98766	98811	98855	98900	98945	98989	99033	99078	45
98	99122	99166	99211	99255	99299	99343	99387	99431	99475	99519	44
99	99563	99607	99651	99694	99738	99782	99825	99869	99913	99956	44

TABLE V.—*Common logarithms of circular functions.*

Arc.	Sine.	Diff.	Cosine.	Diff.	Tang.	Diff.	Cotang.	
° ′								° ′
0 00	Inf. neg.	....	10.00000	....	Inf. neg.	....	Inf. pos.	90 00
01	6.46373	....	10.00000	....	6.46373	....	13.53627	59
02	.76476	30103	10.00000	....	.76476	30103	.23524	58
03	6.94085	17609	10.00000	....	6.94085	17609	13.05915	57
04	7.06579	12494	10.00000	....	7.06579	12494	12.93421	56
05	.16270	9691	10.00000	....	.16270	9691	.83730	55
06	.24188	7918	9.99999	....	.24188	7918	.75812	54
07	.30882	6694	.99999	....	.30882	6694	.69117	53
08	.36682	5800	.99999	....	.36682	5800	.63318	52
09	.41797	5115	.99999	....	.41797	5115	.58203	51
10	.46373	4576	.99999	....	.46373	4576	.53627	50
11	7.50512	4139	9.99999	....	7.50512	4139	12.49488	49
12	.54291	3779	.99999	....	.54291	3779	.45709	48
13	.57767	3476	.99999	....	.57767	3476	.42233	47
14	.60985	3218	.99999	....	.60986	3219	.39014	46
15	.63982	2996	.99999	....	.63982	2996	.36018	45
16	.66784	2803	.99999	....	.66785	2803	.33215	44
17	.69417	2633	.99999	....	.69418	2633	.30582	43
18	.71900	2482	.99999	....	.71900	2482	.28100	42
19	.74248	2348	.99999	....	.74248	2348	.25752	41
20	.76475	2227	.99999	....	.76476	2228	.23524	40
21	7.78594	2119	9.99999	....	7.78595	2119	12.21405	39
22	.80615	2020	.99999	....	.80615	2020	.19384	38
23	.82545	1930	.99999	....	.82546	1931	.17454	37
24	.84393	1848	.99999	....	.84394	1848	.15606	36
25	.86166	1773	.99999	....	.86167	1773	.13833	35
26	.87869	1703	.99999	....	.87871	1704	.12129	34
27	.89508	1639	.99999	....	.89510	1639	.10490	33
28	.91088	1580	.99999	....	.91089	1579	.08911	32
29	.92612	1524	.99998	....	.92613	1524	.07387	31
30	.94084	1472	.99998	....	.94086	1473	.05914	30
31	7.95508	1424	9.99998	....	7.95510	1424	12.04490	29
32	.96887	1379	.99998	....	.96889	1379	.03111	28
33	.98223	1336	.99998	....	.98225	1336	.01775	27
34	7.99520	1297	.99998	....	7.99522	1297	12.00478	26
35	8.00779	1259	.99998	....	8.00781	1259	11.99219	25
36	.02002	1223	.99998	....	.02004	1223	.97996	24
37	.03192	1190	.99997	....	.03194	1190	.96805	23
38	.04350	1158	.99997	....	.04353	1158	.95647	22
39	.05478	1128	.99997	....	.05481	1128	.94519	21
40	.06578	1100	.99997	....	.06581	1100	.93419	20
41	8.07650	1072	9.99997	....	8.07653	1072	11.92347	19
42	.08696	1046	.99997	....	.08700	1047	.91300	18
43	.09718	1022	.99997	....	.09722	1022	.90278	17
44	.10717	998	.99996	....	.10720	999	.89280	16
45	.11693	976	.99996	....	.11696	976	.88304	15
46	.12647	954	.99996	....	.12651	955	.87349	14
47	.13581	934	.99996	....	.13585	934	.86415	13
48	.14495	914	.99996	....	.14500	915	.85500	12
49	.15391	895	.99996	....	.15395	895	.84605	11
50	.16268	877	.99995	....	.16273	878	.83727	10
51	8.17128	860	9.99995	....	8.17133	860	11.82867	9
52	.17971	843	.99995	....	.17976	843	.82024	8
53	.18798	827	.99995	....	.18804	828	.81196	7
54	.19610	812	.99995	....	.19616	812	.80384	6
55	.20407	797	.99994	....	.20413	797	.79587	5
56	.21189	782	.99994	....	.21195	782	.78805	4
57	.21958	769	.99994	....	.21964	769	.78036	3
58	.22713	755	.99994	....	.22719	755	.77280	2
59	.23456	743	.99994	....	.23462	743	.76538	1
60	8.24185	729	9.99993	....	8.24192	730	11.75808	89 0
	Cosine.	Diff.	Sine.	Diff.	Contag.	Diff.	Tang.	Arc.

TABLE V.—*Common logarithms of circular functions*—Continued.

Arc.	Sine.	Diff.	Cosine.	Diff.	Tang.	Diff.	Cotang.	
° ′								° ′
1 00	8.24186	729	9.99993	....	8.24192	730	11.75808	60
01	.24903	718	.99993	....	.24910	718	.75090	59
02	.25609	706	.99993	....	.25616	706	.74383	58
03	.26304	695	.99993	....	.26311	695	.73688	57
04	.26988	684	.99992	....	.26996	685	.73004	56
05	.27661	673	.99992	....	.27669	673	.72331	55
06	.28324	663	.99992	....	.28332	663	.71668	54
07	.28977	653	.99992	....	.28986	654	.71014	53
08	.29621	644	.99991	....	.29629	643	.70371	52
09	.30255	634	.99991	....	.30263	634	.69737	51
10	.30879	624	.99991	....	.30888	625	.69112	50
11	8.31495	616	9.99991	....	8.31505	617	11.68495	49
12	.32103	608	.99990	....	.32112	607	.67888	48
13	.32702	599	.99990	....	.32711	599	.67289	47
14	.33292	590	.99990	....	.33302	591	.66697	46
15	.33875	583	.99990	....	.33886	584	.66114	45
16	.34450	575	.99989	....	.34461	575	.65539	44
17	.35018	568	.99989	....	.35029	568	.64971	43
18	.35578	560	.99989	....	.35589	560	.64410	42
19	.36131	553	.99988	....	.36143	554	.63857	41
20	.36678	547	.99988	....	.36689	546	.63310	40
21	8.37217	539	9.99988	....	8.37229	540	11.62771	39
22	.37750	533	.99988	....	.37762	533	.62238	38
23	.38276	526	.99987	....	.38289	527	.61711	37
24	.38796	520	.99987	....	.38809	520	.61191	36
25	.39310	514	.99987	....	.39323	514	.60677	35
26	.39818	508	.99986	....	.39831	508	.60168	34
27	.40320	502	.99986	....	.40334	503	.59666	33
28	.40816	496	.99986	....	.40830	496	.59170	32
29	.41307	491	.99985	....	.41321	491	.58679	31
30	.41792	485	.99985	....	.41807	486	.58193	30
31	8.42272	480	9.99985	....	8.42287	480	11.57713	29
32	.42746	474	.99984	....	.42762	475	.57238	28
33	.43216	470	.99984	....	.43231	469	.56768	27
34	.43680	464	.99984	....	.43696	465	.56304	26
35	.44139	459	.99983	....	.44156	460	.55844	25
36	.44594	455	.99983	....	.44611	455	.55389	24
37	.45044	450	.99983	....	.45061	450	.54939	23
38	.45489	445	.99982	....	.45507	446	.54493	22
39	.45930	441	.99982	....	.45948	441	.54052	21
40	.46366	436	.99982	....	.46385	437	.53615	20
41	8.46798	432	9.99981	....	8.46817	432	11.53183	19
42	.47226	428	.99981	....	.47245	428	.52755	18
43	.47650	424	.99980	....	.47669	424	.52331	17
44	.48069	419	.99980	....	.48089	420	.51911	16
45	.48485	416	.99980	....	.48505	416	.51495	15
46	.48896	411	.99979	....	.48917	412	.51083	14
47	.49304	408	.99979	....	.49325	408	.50675	13
48	.49708	404	.99979	....	.49729	404	.50271	12
49	.50108	400	.99978	....	.50130	401	.49870	11
50	.50504	396	.99978	....	.50527	397	.49473	10
51	8.50897	393	9.99977	....	8.50920	393	11.49080	9
52	.51287	390	.99977	....	.51310	390	.48690	8
53	.51673	386	.99976	....	.51696	386	.48304	7
54	.52055	382	.99976	....	.52079	383	.47921	6
55	.52434	379	.99976	....	.52459	380	.47541	5
56	.52810	376	.99975	....	.52835	376	.47165	4
57	.53183	373	.99975	....	.53208	373	.46792	3
58	.53552	369	.99974	....	.53578	370	.46422	2
59	.53919	367	.99974	....	.53945	367	.46055	1
60	8.54282	363	9.99973	....	8.54308	363	11.45692	88 0
	Cosine.	Diff.	Sine.	Diff.	Cotang.	Diff.	Tang.	Arc.

TABLE V.—*Common logarithms of circular functions*—Continued.

Arc.	Sine.	Diff.	Cosine.	Diff.	Tang.	Diff.	Cotang.	
° '								° '
2 00	8.54282	363	9.99978	....	8.54308	363	11.45692	60
01	.54642	360	99973	....	.54669	361	.45331	59
02	.54999	357	.99973	....	.55027	358	.44973	58
03	.55354	355	.99972	....	.55382	355	.44618	57
04	.55705	351	.99972	....	.55734	352	.44266	56
05	.56054	349	.99971	....	.56083	349	.43917	55
06	.56400	346	.99971	....	.56429	346	.43571	54
07	.56743	343	.99970	....	.56773	344	.43227	53
08	.57084	341	.99970	....	.57114	341	.42886	52
09	.57421	337	.99969	....	.57452	338	.42548	51
10	.57757	336	.99969	....	.57788	336	.42212	50
11	8.58089	332	9.99968	....	8.58121	333	11.41879	49
12	.58419	330	.99968	....	.58451	330	.41549	48
13	.58747	328	.99967	....	.58779	328	.41220	47
14	.59072	325	.99967	....	.59105	326	.40895	46
15	.59395	323	.99966	....	.59428	323	.40572	45
16	.59715	320	.99966	....	.59749	321	.40251	44
17	.60033	318	.99965	....	.60068	319	.39932	43
18	.60349	316	.99965	....	.60384	316	.39316	42
19	.60662	313	.99964	....	.60698	314	.39302	41
20	.60973	311	.99964	....	.61009	311	.38991	40
21	8.61282	309	9.99963	....	8.61319	310	11.38681	39
22	.61589	307	.99963	....	.61626	307	.38374	38
23	.61894	305	.99962	....	.61931	305	.38069	37
24	.62196	302	.99962	....	.62234	303	.37766	36
25	.62496	300	.99961	....	.62535	301	.37465	35
26	.62795	299	.99961	....	.62834	299	.37166	34
27	.63091	296	.99960	....	.63131	297	.36869	33
28	.63385	294	.99960	....	.63426	295	.36574	32
29	.63678	293	.99959	....	.63718	292	.36282	31
30	.63968	290	.99959	....	.64009	291	.35991	30
31	8.64256	288	9.99958	....	8.64298	289	11.35702	29
32	.64543	287	.99957	....	.64585	287	.35415	28
33	.64827	284	.99957	....	.64870	285	.35130	27
34	.65110	283	.99956	....	.65154	284	.34846	26
35	.65391	281	.99956	....	.65435	281	.34565	25
36	.65670	279	.99955	....	.65715	280	.34285	24
37	.65947	277	.99955	....	.65993	278	.34007	23
38	.66223	276	.99954	....	.66269	276	.33731	22
39	.66497	274	.99953	....	.66543	274	.33457	21
40	.66769	272	.99953	....	.66816	273	.33184	20
41	8.67039	270	9,99952	....	8.67087	271	11.32913	19
42	.67308	269	.99952	....	.67356	269	.32644	18
43	.67575	267	.99951	....	.67624	268	.32376	17
44	.67840	265	.99951	....	.67890	266	.32110	16
45	.68104	264	.99950	....	.68154	264	.31846	15
46	.68366	262	.99949	....	.68417	263	.31583	14
47	.68627	261	.99949	....	.68678	261	.31322	13
48	.68886	259	.99948	....	.68938	260	.31062	12
49	.69144	258	.99947	....	.69196	258	.30804	11
50	.69400	256	.99947	....	.69453	257	.30547	10
51	8.69654	254	9.99946	....	8.69708	255	11.30292	9
52	.69907	253	.99946	....	.69962	254	.30038	8
53	.70159	252	.99945	....	.70214	252	.29786	7
54	.70409	250	.99944	....	.70465	251	.29535	6
55	.70658	249	.99944	....	.70714	249	.29286	5
56	.70905	247	.99943	....	.70962	248	.29038	4
57	.71151	246	.99942	....	.71208	246	.28792	3
58	.71395	244	.99942	....	.71453	245	.28547	2
59	.71638	243	.99941	....	.71697	244	.28303	1
60	8.71880	242	9.99940	....	8.71940	243	11.28060	87 0
	Cosine.	Diff.	Sine.	Diff.	Cotang.	Diff.	Tang.	Arc.

TABLE V.—*Common logarithms of circular functions*—Continued.

Arc.	Sine.	Diff.	Cosine.	Diff.	Tang.	Diff.	Cotang.	
° ′								° ′
3 00	8.71880	242	9.99940	....	8.71940	243	11.28060	60
01	.72120	240	.99940	....	.72181	241	.27819	59
02	.72359	239	.99939	....	.72420	239	.27579	58
03	.72597	238	.99938	....	.72659	239	.27341	57
04	.72834	237	.99938	....	.72896	237	.27104	56
05	.73069	235	.99937	....	.73132	236	.26869	55
06	.73303	234	.99936	....	.73366	234	.26634	54
07	.73535	232	.99936	....	.73600	234	.26400	53
08	.73767	232	.99935	....	.73832	232	.26168	52
09	.73997	230	.99934	....	.74063	231	.25937	51
10	.74226	229	.99934	....	.74292	229	.25708	50
11	8.74454	228	9.99933	....	8.74521	229	11.25479	49
12	.74680	226	.99932	....	.74748	227	.25252	48
13	.74905	225	.99931	....	.74974	226	.25026	47
14	.75130	225	.99931	....	.75199	225	.24801	46
15	.75353	223	.99930	....	.75423	224	.24577	45
16	.75575	222	.99929	....	.75645	222	.24355	44
17	.75795	220	.99929	....	.75867	222	.24133	43
18	.76015	220	.99928	....	.76087	220	.23913	42
19	.76234	219	.99927	....	.76306	219	.23693	41
20	.76451	217	.99926	....	.76525	219	.23475	40
21	8.76667	216	9.99926	....	8.76742	217	11.23258	39
22	.76883	216	.99926	....	.76958	216	.23042	38
23	.77097	214	.99924	....	.77173	215	.22827	37
24	.77310	213	.99923	....	.77387	214	.22613	36
25	.77522	212	.99923	....	.77599	212	.22400	35
26	.77733	211	.99922	....	.77811	212	.22189	34
27	.77943	210	.99921	....	.78022	211	.21978	33
28	.78152	209	.99920	....	.78232	210	.21768	32
29	.78360	208	.99920	....	.78441	209	.21559	31
30	.78567	207	.99919	....	.78649	208	.21351	30
31	8.78774	207	9.99918	....	8.78855	206	11.21145	29
32	.78979	205	.99917	....	.79061	206	.20939	28
33	.79183	204	.99917	....	.79266	205	.20734	27
34	.79386	203	.99916	....	.79470	204	.20530	26
35	.79588	202	.99915	....	.79673	203	.20327	25
36	.79789	201	.99914	....	.79875	202	.20125	24
37	.79990	201	.99913	....	.80076	201	.19924	23
38	.80189	199	.99913	....	.80276	200	.19723	22
39	.80388	199	.99912	....	.80476	200	.19524	21
40	.80585	197	.99911	....	.80674	198	.19326	20
41	8.80782	197	9.99910	....	8.80872	198	11.19128	19
42	.80978	196	.99909	....	.81068	196	.18932	18
43	.81173	195	.99909	....	.81264	196	.18736	17
44	.81367	194	.99908	....	.81459	195	.18541	16
45	.81560	193	.99907	....	.81653	194	.18347	15
46	.81752	192	.99906	....	.81846	193	.18154	14
47	.81944	192	.99905	....	.82038	192	.17962	13
48	.82134	190	.99904	....	.82230	192	.17770	12
49	.82324	190	.99904	....	.82420	190	.17579	11
50	.82513	189	.99903	....	.82610	190	.17390	10
51	8.82701	188	9.99902	....	8.82799	189	11.17201	9
52	.82888	187	.99901	....	.82987	188	.17013	8
53	.83075	187	.99900	....	.83175	188	.16825	7
54	.83261	186	.99899	....	.83361	186	.16639	6
55	.83446	185	.99898	....	.83547	186	.16453	5
56	.83630	184	.99898	....	.83732	185	.16268	4
57	.83813	183	.99897	....	.83916	184	.16084	3
58	.83996	183	.99896	....	.84100	184	.15900	2
59	.84177	181	.99895	....	.84282	182	.15717	1
60	8.84358	181	9.99894	....	8.84464	182	11.15536	86 0
	Cosine.	Diff.	Sine.	Diff.	Cotang.	Diff.	Tang.	Arc.

TABLE V.—*Common logarithms of circular functions*—Continued.

Arc.		Sine.	Diff.	Cosine.	Diff.	Tang.	Diff.	Cotang.		
°	'								°	'
4	00	8.84358	181	9.99894	1	8.84464	182	11.15536	86	00
	10	.86128	1770	.99885	9	.86243	1779	.13757		50
	20	.87828	1700	.99876	9	.87953	1710	.12047		40
	30	.89464	1636	.99866	10	.89598	1645	.10402		30
	40	.91040	1576	.99856	10	.91185	1587	.08815		20
	50	.92561	1521	.99845	11	.92716	1531	.07284		10
5	00	8.94030	1469	9.99834	11	8.94195	1479	11.05805	85	00
	10	.95450	1420	.99823	11	.95627	1432	.04373		50
	20	.96825	1375	.99812	11	.97013	1386	.02987		40
	30	.98157	1332	.99800	12	.98358	1345	.01642		30
	40	.99450	1293	.99787	13	.99662	1304	.00338		20
	50	9.00704	1254	.99774	13	9.00930	1268	10.99070		10
6	00	9.01923	1219	9.99761	13	9.02162	1232	10.97838	84	00
	10	.03109	1186	.99748	13	.03361	1199	.96639		50
	20	.04262	1153	.99734	14	.04528	1167	.95472		40
	30	.05386	1124	.99720	14	.05666	1138	.94334		30
	40	.06481	1095	.99705	15	.06775	1109	.93225		20
	50	.07548	1067	.99690	15	.07858	1083	.92142		10
7	00	9.08589	1041	9.99675	15	9.08914	1056	10.91086	83	00
	10	.09606	1017	.99659	16	.09947	1033	.90053		50
	20	.10599	993	.99643	16	.10956	1009	.89044		40
	30	.11570	971	.99627	16	.11943	987	.88057		30
	40	.12519	949	.99610	17	.12909	966	.87091		20
	50	.13447	928	.99593	17	.13854	945	.86146		10
8	00	9.14355	908	9.99575	18	9.14780	926	10.85220	82	00
	10	.15245	890	.99557	18	.15688	908	.84312		50
	20	.16116	871	.99539	18	.16577	889	.83423		40
	30	.16970	854	.99520	19	.17450	873	.82550		30
	40	.17807	837	.99501	19	.18306	856	.81694		20
	50	.18628	821	.99482	19	.19146	840	.80854		10
9	00	9.19433	805	9.99462	20	9.19971	825	10.80029	81	00
	10	.20223	790	.99442	20	.20782	811	.79218		50
	20	.20999	776	.99421	21	.21578	796	.78422		40
	30	.21761	762	.99400	21	.22361	783	.77639		30
	40	.22509	748	.99379	21	.23130	769	.76870		20
	50	.23244	735	.99357	22	.23887	757	.76113		10
10	00	9.23967	723	9.99335	22	9.24632	745	10.75368	80	00
	10	.24677	710	.99313	22	.25365	733	.74635		50
	20	.25376	699	.99290	23	.26086	721	.73914		40
	30	.26063	687	.99267	23	.26797	711	.73203		30
	40	.26739	676	.99243	24	.27496	699	.72504		20
	50	.27405	666	.99219	24	.28186	690	.71814		10
11	00	9.28060	655	9.99195	24	9.28865	679	10.71135	79	00
	10	.28705	645	.99170	25	.29535	670	.70465		50
	20	.29340	635	.99145	25	.30195	660	.69805		40
	30	.29965	625	.99119	26	.30846	651	.69154		30
	40	.30582	617	.99093	26	.31488	642	.68511		20
	50	.31189	607	.99067	26	.32122	634	.67878		10
12	00	9.31788	599	9.99040	27	9.32747	625	10.67252	78	00
	10	.32378	590	.99013	27	.33365	618	.66635		50
	20	.32960	582	.98986	27	.33974	609	.66026		40
	30	.33534	574	.98958	28	.34575	601	.65424		30
	40	.34100	566	.98930	28	.35170	595	.64830		20
	50	.34658	558	.98901	29	.35757	587	.64243		10
		Cosine.	Diff.	Sine.	Diff.	Cotang.	Diff.	Tang.	Arc.	

TABLE V.—*Common logarithms of circular functions*—Continued.

Arc.		Sine.	Diff.	Cosine.	Diff.	Tang.	Diff.	Cotang.		
°	′								°	′
13	00	9.35209	551	9.98872	29	9.36336	579	10.63664	77	00
	10	.35752	543	.98843	29	.36909	573	.63091		50
	20	.36289	537	.98813	30	.37476	567	.62524		40
	30	.36818	528	.98783	30	.38035	559	.61965		30
	40	.37341	523	.98753	30	.38589	554	.61411		20
	50	.37858	517	.98722	31	.39136	547	.60864		10
14	00	9.38367	509	9.98690	32	9.39677	541	10.60323	76	00
	10	.38871	504	.98659	31	.40212	535	.59788		50
	20	.39368	497	.98627	32	.40742	530	.59258		40
	30	.39860	492	.98594	33	.41266	524	.58734		30
	40	.40345	485	.98561	33	.41784	518	.58216		20
	50	.40825	480	.98528	33	.42297	513	.57703		10
15	00	9.41300	475	9.98494	34	9.42805	508	10.57195	75	00
	10	.41768	468	.98460	34	.43308	503	.56692		50
	20	.42232	464	.98426	34	.43806	498	.56194		40
	30	.42690	458	.98391	35	.44299	493	.55701		30
	40	.43143	453	.98356	35	.44787	488	.55213		20
	50	.43591	448	.98320	36	.45271	484	.54729		10
16	00	9.44034	443	9.98284	36	9.45750	479	10.54250	74	00
	10	.44472	438	.98248	36	.46224	474	.53776		50
	20	.44905	433	.98211	37	.46694	470	.53305		40
	30	.45334	429	.98174	37	.47160	466	.52839		30
	40	.45758	424	.98136	38	.47622	462	.52378		20
	50	.46178	420	.98098	38	.48080	458	.51920		10
17	00	9.46593	415	9.98060	38	9.48534	454	10.51466	73	00
	10	.47005	412	.98021	39	.48984	450	.51016		50
	20	.47411	406	.97982	39	.49430	446	.50570		40
	30	.47814	403	.97942	40	.49872	442	.50128		30
	40	.48213	399	.97902	40	.50311	439	.49689		20
	50	.48607	394	.97861	41	.50746	435	.49254		10
18	00	9.48998	391	9.97821	40	9.51178	432	10.48822	72	00
	10	.49385	387	.97779	42	.51606	428	.48394		50
	20	.49768	383	.97738	41	.52030	424	.47969		40
	30	.50148	380	.97696	42	.52452	422	.47548		30
	40	.50523	375	.97653	43	.52870	418	.47130		20
	50	.50896	373	.97610	43	.53285	415	.46715		10
19	00	9.51264	368	9.97567	43	9.53697	412	10.46303	71	00
	10	.51629	365	.97523	44	.54106	409	.45894		50
	20	.51991	362	.97479	44	.54512	406	.45488		40
	30	.52349	358	.97435	44	.54915	403	.45085		30
	40	.52705	356	.97390	45	.55315	400	.44685		20
	50	.53056	351	.97344	46	.55712	397	.44288		10
20	00	9.53405	349	9.97299	45	9.56107	395	10.43893	70	00
	10	.53751	346	.97252	47	.56498	391	.43502		50
	20	.54093	342	.97206	46	.56887	389	.43113		40
	30	.54432	339	.97159	47	.57274	387	.42726		30
	40	.54769	337	.97111	48	.57658	384	.42342		20
	50	.55102	333	.97063	48	.58039	381	.41961		10
21	00	9.55433	331	9.97015	48	9.58418	379	10.41582	69	00
	10	.55761	328	.96966	49	.58794	376	.41206		50
	20	.56085	324	.96917	49	.59168	374	.40832		40
	30	.56407	322	.96868	49	.59540	372	.40460		30
	40	.56727	320	.96818	50	.59909	369	.40091		20
	50	.57043	316	.96767	51	.60276	367	.39724		10
		Cosine.	Diff.	Sine.	Diff.	Cotang.	Diff.	Tang.	Arc.	

TABLE V.—*Common logarithms of circular functions*—Continued.

Arc.		Sine.	Diff.	Cosine.	Diff.	Tang.	Diff.	Cotang.		
°	'								°	'
22	00	9.57357	314	9.96717	50	9.60641	365	10.39359	68	00
	10	.57669	312	.96665	52	.61004	363	.38996		50
	20	.57978	309	.96614	51	.61364	360	.38636		40
	30	.58284	306	.96561	53	.61722	358	.38278		30
	40	.58588	304	.96509	52	.62079	357	.37921		20
	50	.58889	301	.96456	53	.62433	354	.37567		10
23	00	9.59188	299	9.96403	53	9.62785	352	10.37215	67	00
	10	.59484	296	.96349	54	.63135	350	.36864		50
	20	.59778	294	.96294	55	.63484	349	.36516		40
	30	.60070	292	.96240	54	.63830	346	.36170		30
	40	.60359	289	.96185	55	.64175	345	.35825		20
	50	.60646	287	.96129	56	.64517	342	.35483		10
24	00	9.60931	285	9.96073	56	9.64858	341	10.35142	66	00
	10	.61214	283	.96016	57	.65197	339	.34803		50
	20	.61494	280	.95960	56	.65535	338	.34465		40
	30	.61773	279	.95902	58	.65870	335	.34130		30
	40	.62049	276	.95844	58	.66204	334	.33796		20
	50	.62323	274	.95786	58	.66537	333	.33463		10
25	00	9.62595	272	9.95728	58	9.66867	330	10.33133	65	00
	10	.62865	270	.95668	60	.67196	329	.32804		50
	20	.63133	268	.95609	59	.67524	328	.32476		40
	30	.63398	265	.95549	60	.67850	326	.32150		30
	40	.63662	264	.95488	61	.68174	324	.31826		20
	50	.63924	262	.95427	61	.68497	323	.31503		10
26	00	9.64184	260	9.95366	60	9.68818	321	10.31182	64	00
	10	.64442	258	.95304	62	.69138	320	.30862		50
	20	.64698	256	.95242	62	.69457	319	.30543		40
	30	.64953	255	.95179	63	.69774	317	.30226		30
	40	.65205	252	.95116	63	.70089	315	.29911		20
	50	.65456	251	.95052	64	.70404	315	.29596		10
27	00	9.65705	249	9.94988	64	9.70717	313	10.29283	63	00
	10	.65952	247	.94923	65	.71028	311	.28972		50
	20	.66197	245	.94858	65	.71339	311	.28661		40
	30	.66441	244	.94793	65	.71648	309	.28352		30
	40	.66682	241	.94727	66	.71955	307	.28044		20
	50	.66922	240	.94660	67	.72262	307	.27738		10
28	00	9.67161	239	9.94593	67	9.72567	305	10.27433	62	00
	10	.67398	237	.94526	67	.72872	305	.27128		50
	20	.67633	235	.94458	68	.73175	303	.26825		40
	30	.67866	233	.94390	68	.73476	301	.26524		30
	40	.68098	232	.94321	69	.73777	301	.26223		20
	50	.68328	230	.94252	69	.74077	300	.25923		10
29	00	9.68557	229	9.94182	70	9.74375	298	10.25625	61	00
	10	.68784	227	.94112	70	.74673	298	.25327		50
	20	.69010	226	.94041	71	.74969	296	.25031		40
	30	.69234	224	.93970	71	.75264	295	.24736		30
	40	.69456	222	.93898	72	.75558	294	.24441		20
	50	.69677	221	.93826	72	.75852	294	.24148		10
30	00	9.69897	220	9.93753	73	9.76144	292	10.23856	60	00
	10	.70115	218	.93680	73	.76435	291	.23565		50
	20	.70332	217	.93606	74	.76725	290	.22274		40
	30	.70547	215	.93532	74	.77015	290	.22985		30
	40	.70761	214	.93457	75	.77303	288	.22697		20
	50	.70973	212	.93382	75	.77591	288	.22409		10
		Cosine.	Diff.	Sine.	Diff.	Cotang.	Diff.	Tang.	Arc.	

TABLE V.—*Common logarithms of circular functions*—Continued.

Arc. °	'	Sine.	Diff.	Cosine.	Diff.	Tang.	Diff.	Cotang.	°	'
31	00	9.71184	211	9.93307	75	9.77877	286	10.22123	59	00
	10	.71393	209	.93230	77	.78163	286	.21837		50
	20	.71602	209	.93154	76	.78448	285	.21552		40
	30	.71808	206	.93077	77	.78732	284	.21268		30
	40	.72014	206	.92999	78	.79015	283	.20985		20
	50	.72218	204	.92921	78	.79297	282	.20703		10
32	00	9.72421	203	9.92842	79	9.79579	282	10.20421	58	00
	10	.72622	201	.92763	79	.79860	281	.20140		50
	20	.72823	201	.92683	80	.80140	280	.19860		40
	30	.73022	199	.92603	80	.80419	279	.19581		30
	40	.73219	197	.92522	81	.80697	278	.19303		20
	50	.73416	197	.92441	81	.80975	278	.19025		10
33	00	9.73611	195	9.92359	82	9.81252	277	10.18748	57	00
	10	.73805	194	.92277	82	.81528	276	.18472		50
	20	.73997	192	.92194	83	.81803	275	.18196		40
	30	.74189	192	.92111	83	.82078	275	.17922		30
	40	.74379	190	.92027	84	.82352	274	.17648		20
	30	.74568	189	.91942	85	.82626	274	.17374		10
34	00	9.74756	188	9.91857	85	9.82899	273	10.17101	56	00
	10	.74943	187	.91772	85	.83171	272	.16829		50
	20	.75128	185	.91686	86	.83442	271	.16557		40
	30	.75313	185	.91599	87	.83713	271	.16287		30
	40	.75496	183	.91512	87	.83984	271	.16016		20
	50	.75678	182	.91425	87	.84253	269	.15746		10
35	00	9.75859	181	9.91336	89	9.84523	270	10.15477	55	00
	10	.76039	180	.91248	88	.84791	268	.15209		50
	20	.76218	179	.91158	90	.85059	268	.14941		40
	30	.76395	177	.91069	89	.85327	268	.14673		30
	40	.76572	177	.90978	91	.85594	267	.14406		20
	50	.76747	175	.90887	91	.85860	266	.14140		10
36	00	9.76922	175	9.90796	91	9.86126	266	10.13874	54	00
	10	.77095	173	.90704	92	.86391	265	.13608		50
	20	.77267	172	.90611	93	.86656	265	.13344		40
	30	.77439	172	.90518	93	.86921	265	.13079		30
	40	.77609	170	.90424	94	.87185	264	.12815		20
	50	.77778	169	.90330	94	.87448	263	.12552		10
37	00	9.77946	168	9.90235	95	9.87711	263	10.12289	53	00
	10	.78113	167	.90139	96	.87974	263	.12026		50
	20	.78280	167	.90043	96	.88236	262	.11764		40
	30	.78445	165	.89947	96	.88498	262	.11502		30
	40	.78609	164	.89849	98	.88759	261	.11241		20
	50	.78772	163	.89752	97	.89020	261	.10980		10
38	00	9.78934	162	9.89653	99	9.89281	261	10.10719	52	00
	10	.79095	161	.89554	99	.89541	260	.10459		50
	20	.79256	161	.89455	99	.89801	260	.10199		40
	30	.79415	159	.89354	101	.90060	259	.09939		30
	40	.79573	158	.89254	100	.90320	260	.09680		20
	50	.79731	158	.89152	102	.90578	258	.09421		10
39	00	9.79887	156	9.89050	102	9.90837	259	10.09163	51	00
	10	.80043	156	.88948	102	.91095	258	.08905		50
	20	.80197	154	.88844	104	.91353	258	.08647		40
	30	.80351	154	.88741	103	.91610	257	.08390		30
	40	.80504	153	.88636	105	.91868	258	.08132		20
	50	.80656	152	.88531	105	.92125	257	.07875		10
		Cosine.	Diff.	Sine.	Diff.	Cotang.	Diff.	Tang.	Arc.	

TABLE V.—*Common logarithms of circular functions*—Continued.

Arc.	Sine.	Diff.	Cosine.	Diff.	Tang.	Diff.	Cotang.	
° ′								° ′
40 00	9.80807	151	9.88425	106	9.92381	256	10.07618	50 00
10	.80957	150	.88319	106	.92638	257	.07362	50
20	.81106	149	.88212	107	.92894	256	.07106	40
30	.81254	148	.88105	107	.93150	256	.06850	30
40	.81402	148	.87996	109	.93406	256	.06594	20
50	.81548	146	.87887	109	.93661	255	.06339	10
41 00	9.81694	146	9.87778	109	9.93916	255	10.06084	49 00
10	.81839	145	.87668	110	.94171	255	.05829	50
20	.81983	144	.87557	111	.94426	255	.05574	40
30	.82126	143	.87446	111	.94681	255	.05319	30
40	.82269	143	.87333	113	.94935	254	.05065	20
50	.82410	141	.87221	112	.95190	255	.04817	10
42 00	9.82551	141	9.87107	114	9.95444	254	10.04556	48 00
10	.82691	140	.86993	114	.95698	254	.04302	50
20	.82830	139	.86878	115	.95952	254	.04048	40
30	.82968	138	.86763	115	.96205	253	.03795	30
40	.83106	138	.86647	116	.96459	254	.03541	20
50	.83242	136	.86530	117	.96712	253	.03288	10
43 00	9.83378	136	9.86413	117	9.96966	254	10.03034	47 00
10	.83513	135	.86295	118	.97219	253	.02781	50
20	.83648	135	.86176	119	.97472	253	.02528	40
30	.83781	133	.86056	120	.97725	253	.02275	30
40	.83914	133	.85936	120	.97978	253	.02022	20
50	.84046	132	.85815	121	.98231	253	.01769	10
44 00	9.84177	131	9.856[illegible]	122	9.98484	253	10.01516	46 00
10	.84308	131	.85571	122	.98736	252	.01263	50
20	.84437	129	.85448	123	.98989	253	.01011	40
30	.84566	129	.85324	124	.99242	253	.00758	30
40	.84694	128	.85200	124	.99495	253	.00505	20
50	.84822	128	.85074	126	.99747	252	.00253	10
45 00	9.84948	126	9.84948	126	10.00000	253	10.00000	45 00
	Cosine.	Diff.	Sine.	Diff.	Cotang.	Diff.	Tang.	Arc.

212. **The slide rule** is a contrivance for using logs. mechanically. It consists, figure 47, of a rule, in the middle of which is a slide. The edges of the groove and the edges of the slide are graduated, forming four scales called A, B, C, and D. An indicator, which can be set at any point, guides the eye in selecting opposite numbers. The slide rule deals with **mantissas** only. Characteristics must be obtained by inspection.

To multiply.—Move the slide to the *right* until 1 on scale B is opposite the smaller of the two numbers on A; the number on A opposite the larger of the two numbers on B is the product.

To divide.—Move the slide to the *left* until the divisor on B is under 1 on A. The number on A opposite the *dividend* on B is the quotient desired. **To multiply and divide simultaneously**, or to **solve a proportion**, set the divisor on B opposite one of the other numbers on A. The number on A opposite the third number on B is the result desired.

To find the square of a number.—Take the number on A opposite the given number on D.

To find the square root.—Take the number on D opposite the given number on A. In taking square roots use only the *left half* of A, for an odd number of figures in front of the decimal point, and the *right half* only for *even* number.

To find a cube.—Set 1 on B opposite the given number on D. The number on A opposite the given number on B is the cube desired.

To find a cube root.—Take the root approximately by inspection. Set this number on B opposite the given number on A. Note whether 1 on C is opposite the approximate root on D. If so, the approximate root is the correct one; if not, move the slide slightly one way or the other until the number on B opposite the given number, and the number on D opposite the one on C are the same. This number is the desired cube root.

Occasional users of the slide rule will do well to adhere to the simple operations above described. Regular users will study the theory and scope of the rule from one of the several treatises on the subject.

TABLE VI.—*Table of squares, cubes, square roots, and cube roots of numbers from 1 to 1,000.*

No.	Square.	Cube.	Square root.	Cube root.	No.	Square.	Cube.	Square root.	Cube root.
1	1	1	1.	1.	57	3249	185193	7.5498	3.8485
2	4	8	1.4142	1.2599	58	3364	195112	7.6158	3.8709
3	9	27	1.7321	1.4422	59	3481	205379	7.6811	3.8930
4	16	64	2.0000	1.5874	60	3600	216000	7.7460	3.9149
5	25	125	2.2361	1.7100	61	3721	226981	7.8102	3.9365
6	36	216	2.4495	1.8171	62	3844	238328	7.8740	3.9579
7	49	343	2.6458	1.9129	63	3969	250047	7.9373	3.9791
8	64	512	2.8284	2.0000	64	4096	262144	8.	4.
9	81	729	3.0000	2.0801	65	4225	274625	8.0623	4.0207
10	100	1000	3.1623	2.1544	66	4356	287496	8.1240	4.0412
11	121	1331	3.3166	2.2240	67	4489	300763	8.1854	4.0615
12	144	1728	3.4641	2.2894	68	4624	314432	8.2462	4.0817
13	169	2197	3.6056	2.3513	69	4761	328509	8.3066	4.1016
14	196	2744	3.7417	2.4101	70	4900	343000	8.3666	4.1213
15	225	3375	3.8730	2.4662	71	5041	357911	8.4261	4.1408
16	256	4096	4.	2.5198	72	5184	373248	8.4853	4.1602
17	289	4913	4.1231	2.5713	73	5329	389017	8.5440	4.1793
18	324	5832	4.2426	2.6207	74	5476	405224	8.6023	4.1983
19	361	6859	4.3589	2.6684	75	5625	421875	8.6603	4.2172
20	400	8000	4.4721	2.7144	76	5776	438976	8.7178	4.2358
21	441	9261	4.5826	2.7589	77	5929	456533	8.7750	4.2543
22	484	10648	4.6904	2.8020	78	6084	474552	8.8318	4.2727
23	529	12167	4.7958	2.8439	79	6241	493039	8.8882	4.2908
24	576	13824	4.8990	2.8845	80	6400	512000	8.9443	4.3089
25	625	15625	5.	2.9240	81	6561	531441	9.	4.3267
26	676	17576	5.0990	2.9625	82	6724	551368	9.0554	4.3445
27	729	19683	5.1962	3.0000	83	6889	571787	9.1104	4.3621
28	784	21952	5.2915	3.0366	84	7056	592704	9.1652	4.3795
29	841	24389	5.3852	3.0723	85	7225	614125	9.2195	4.3968
30	900	27000	5.4772	3.1072	86	7396	636056	9.2736	4.4140
31	961	29791	5.5678	3.1414	87	7569	658503	9.3274	4.4310
32	1024	32768	5.6569	3.1748	88	7744	681472	9.3808	4.4480
33	1089	35937	5.7446	3.2075	89	7921	704969	9.4340	4.4647
34	1156	39304	5.8310	3.2396	90	8100	729000	9.4868	4.4814
35	1225	42875	5.9161	3.2711	91	8281	753571	9.5394	4.4979
36	1296	46656	6.	3.3019	92	8464	778688	9.5917	4.5144
37	1369	50653	6.0828	3.3322	93	8649	804357	9.6437	4.5307
38	1444	54872	6.1644	3.3620	94	8836	830584	9.6954	4.5468
39	1521	59319	6.2450	3.3912	95	9025	857375	9.7468	4.5629
40	1600	64000	6.3246	3.4200	96	9216	884736	9.7980	4.5789
41	1681	68921	6.4031	3.4482	97	9409	912673	9.8489	4.5947
42	1764	74088	6.4807	3.4760	98	9604	941192	9.8995	4.6104
43	1849	79507	6.5574	3.5034	99	9801	970299	9.9499	4.6261
44	1936	85184	6.6332	3.5303	100	10000	1000000	10.	4.6416
45	2025	91125	6.7082	3.5569	101	10201	1030301	10.0499	4.6570
46	2116	97336	6.7823	3.5830	102	10404	1061208	10.0995	4.6725
47	2209	103823	6.8557	3.6088	103	10609	1092727	10.1489	4.6875
48	2304	110592	6.9282	3.6342	104	10816	1124864	10.1980	4.7027
49	2401	117649	7.	3.6593	105	11025	1157625	10.2470	4.7177
50	2500	125000	7.0711	3.6840	106	11236	1191016	10.2956	4.7326
51	2601	132651	7.1414	3.7084	107	11449	1225043	10.3441	4.7475
52	2704	140608	7.2111	3.7325	108	11664	1259712	10.3923	4.7622
53	2809	148877	7.2801	3.7563	109	11881	1295029	10.4403	4.7769
54	2916	157464	7.3485	3.7798	110	12100	1331000	10.4881	4.7914
55	3025	166375	7.4162	3.8030	111	12321	1367631	10.5357	4.8059
56	3136	175616	7.4833	3.8259	112	12544	1404928	10.5830	4.8203

TABLE VI.—*Table of squares, cubes, square roots, and cube roots of numbers from 1 to 1,000*—Continued.

No.	Square.	Cube.	Square root.	Cube root.	No.	Square.	Cube.	Square root.	Cube root.
113	12769	1442897	10.6301	4.8346	192	36864	7077888	13.8564	5.7690
114	12996	1481544	10.6771	4.8488	193	37249	7189057	13.8924	5.7790
115	13225	1520875	10.7238	4.8629	194	37636	7301384	13.9284	5.7890
116	13456	1560896	10.7703	4.8770	195	38025	7414875	13.9642	5.7989
117	13689	1601613	10.8167	4.8910	196	38416	7529536	14.	5.8088
118	13924	1643032	10.8628	4.9049	197	38809	7645373	14.0357	5.8186
119	14161	1685159	10.9087	4.9187	198	39204	7762392	14.0712	5.8285
120	14400	1728000	10.9545	4.9324	199	39601	7880599	14.1067	5.8383
121	14641	1771561	11.	4.9461	200	40000	8000000	14.1421	5.8480
122	14884	1815848	11.0454	4.9597	201	40401	8120601	14.1774	5.8578
123	15129	1860867	11.0905	4.9732	202	40804	8242408	14.2127	5.8675
124	15376	1906624	11.1355	4.9866	203	41209	8365427	14.2478	5.8771
125	15625	1953125	11.1803	5.	204	41616	8489664	14.2829	5.8868
126	15876	2000376	11.2250	5.0133	205	42025	8615125	14.3178	5.8964
127	16129	2048383	11.2694	5.0265	206	42436	8741816	14.3527	5.9059
128	16384	2097152	11.3137	5.0397	207	42849	8869743	14.3875	5.9155
129	16641	2146689	11.3578	5.0528	208	43264	8998912	14.4222	5.9250
130	16900	2197000	11.4018	5.0658	209	43681	9129329	14.4568	5.9345
131	17161	2248091	11.4455	5.0788	210	44100	9261000	14.4914	5.9439
132	17424	2299968	11.4891	5.0916	211	44521	9393931	14.5258	5.9533
133	17689	2352637	11.5326	5.1045	212	44944	9528128	14.5602	5.9627
134	17956	2406104	11.5758	5.1172	213	45369	9663597	14.5945	5.9721
135	18225	2460375	11.6190	5.1299	214	45796	9800344	14.6287	5.9814
136	18496	2515456	11.6619	5.1426	215	46225	9938375	14.6629	5.9907
137	18769	2571353	11.7047	5.1551	216	46656	10077696	14.6969	6
138	19044	2628072	11.7473	5.1676	217	47089	10218313	14.7309	6.0092
139	19321	2685619	11.7898	5.1801	218	47524	10360232	14.7648	6.0185
140	19600	2744000	11.8322	5.1925	219	47961	10503459	14.7986	6.0277
141	19881	2803221	11.8743	5.2048	220	48400	10648000	14.8324	6.0368
142	20164	2863288	11.9164	5.2171	221	48841	10793861	14.8661	6.0459
143	20449	2924207	11.9583	5.2293	222	49284	10941048	14.8997	6.0550
144	20736	2985984	12.	5.2415	223	49729	11089567	14.9332	6.0641
145	21025	3048625	12.0416	5.2536	224	50176	11239424	14.9666	6.0732
146	21316	3112136	12.0830	5.2656	225	50625	11390625	15.	6.0822
147	21609	3176523	12.1244	5.2776	226	51076	11543176	15.0333	6.0912
148	21904	3241792	12.1655	5.2896	227	51529	11697083	15.0665	6.1002
149	22201	3307949	12.2066	5.3015	228	51984	11852352	15.0997	6.1091
150	22500	3375000	12.2474	5.3133	229	52441	12008989	15.1327	6.1180
151	22801	3442951	12.2882	5.3251	230	52900	12167000	15.1658	6.1269
152	23104	3511808	12.3288	5.3368	231	53361	12326391	15.1987	6.1358
153	23409	3581577	12.3693	5.3485	232	53824	12487168	15.2315	6.1446
154	23716	3652264	12.4097	5.3601	233	54289	12649337	15.2643	6.1534
155	24025	3723875	12.4499	5.3717	234	54756	12812904	15.2971	6.1622
156	24336	3796416	12.4900	5.3832	235	55225	12977875	15.3297	6.1710
157	24649	3869893	12.5300	5.3947	236	55696	13144256	15.3623	6.1797
158	24964	3944312	12.5698	5.4061	237	56169	13312053	15.3948	6.1885
159	25281	4019679	12.6095	5.4175	238	56644	13481272	15.4272	6.1972
160	25600	4096000	12.6491	5.4288	239	57121	13651919	15.4596	6.2058
161	25921	4173281	12.6886	5.4401	240	57600	13824000	15.4919	6.2145
162	26244	4251528	12.7279	5.4514	241	58081	13997521	15.5242	6.2231
163	26569	4330747	12.7671	5.4626	242	58564	14172488	15.5563	6.2317
164	26896	4410944	12.8062	5.4737	243	59049	14348907	15.5885	6.2403
165	27225	4492125	12.8452	5.4848	244	59536	14526784	15.6205	6.2488
166	27556	4574296	12.8841	5.4959	245	60025	14706125	15.6525	6.2573
167	27889	4657463	12.9228	5.5069	246	60516	14886936	15.6844	6.2658
168	28224	4741632	12.9615	5.5178	247	61009	15069223	15.7162	6.2743
169	28561	4826809	13.	5.5288	248	61504	15252992	15.7480	6.2828
170	28900	4913000	13.0384	5.5397	249	62001	15438249	15.7797	6.2912
171	29241	5000211	13.0767	5.5505	250	62500	15625000	15.8114	6.2996
172	29584	5088448	13.1149	5.5613	251	63001	15813251	15.8430	6.3080
173	29929	5177717	13.1529	5.5721	252	63504	16003008	15.8745	6.3164
174	30276	5268024	13.1909	5.5828	253	64009	16194277	15.9060	6.3247
175	30625	5359375	13.2288	5.5934	254	64516	16387064	15.9374	6.3330
176	30976	5451776	13.2665	5.6041	255	65025	16581375	15.9687	6.3413
177	31329	5545233	13.3041	5.6147	256	65536	16777216	16	6.3496
178	31684	5639752	13.3417	5.6252	257	66049	16974593	16.0312	6.3579
179	32041	5735339	13.3791	5.6357	258	66564	17173512	16.0624	6.3661
180	32400	5832000	13.4164	5.6462	259	67081	17373979	16.0935	6.3743
181	32761	5929741	13.4536	5.6567	260	67600	17576000	16.1245	6.3825
182	33124	6028568	13.4907	5.6671	261	68121	17779581	16.1555	6.3907
183	33489	6128487	13.5277	5.6774	262	68644	17984728	16.1864	6.3988
184	33856	6229504	13.5647	5.6877	263	69169	18191447	16.2173	6.4070
185	34225	6331625	13.6015	5.6980	264	69696	18399744	16.2481	6.4151
186	34596	6434856	13.6382	5.7083	265	70225	18609625	16.2788	6.4232
187	34969	6539203	13.6748	5.7185	266	70756	18821096	16.3095	6.4312
188	35344	6644672	13.7113	5.7287	267	71289	19034163	16.3401	6.4393
189	35721	6751269	13.7477	5.7388	268	71824	19248832	16.3707	6.4473
190	36100	6859000	13.7840	5.7489	269	72361	19465109	16.4012	6.4553
191	36481	6967871	13.8203	5.7590	270	72900	19683000	16.4317	6.4633

TABLE VI.—*Table of squares, cubes, square roots, and cube roots of numbers from 1 to 1,000*—Continued.

No.	Square.	Cube.	Square root.	Cube root.	No.	Square.	Cube.	Square root.	Cube root.
271	73441	19902511	16.4621	6.4713	350	122500	42875000	18.7083	7.0473
272	73984	20123648	16.4924	6.4792	351	123201	43243551	18.7350	7.0540
273	74529	20346417	16.5227	6.4872	352	123904	43614208	18.7617	7.0607
274	75076	20570824	16.5529	6.4951	353	124609	43986977	18.7883	7.0674
275	75625	20796875	16.5831	6.5030	354	125316	44361864	18.8149	7.0740
276	76176	21024576	16.6132	6.5108	355	126025	44738875	18.8414	7.0807
277	76729	21253933	16.6433	6.5187	356	126736	45118016	18.8680	7.0873
278	77284	21484952	16.6733	6.5265	357	127449	45499293	18.8944	7.0940
279	77841	21717639	16.7033	6.5343	358	128164	45882712	18.9209	7.1006
280	78400	21952000	16.7332	6.5421	359	128881	46268279	18.9473	7.1072
281	78961	22188041	16.7631	6.5499	360	129600	46656000	18.9737	7.1138
282	79524	22425768	16.7929	6.5577	361	130321	47045881	19.	7.1204
283	80089	22665187	16.8226	6.5654	362	131044	47437928	19.0263	7.1269
284	80656	22906304	16.8523	6.5731	363	131769	47832147	19.0526	7.1335
285	81225	23149125	16.8819	6.5808	364	132496	48228544	19.0788	7.1400
286	81796	23393656	16.9115	6.5885	365	133225	48627125	19.1050	7.1466
287	82369	23639903	16.9411	6.5962	366	133956	49027896	19.1311	7.1531
288	82944	23887872	16.9706	6.6039	367	134689	49430863	19.1572	7.1596
289	83521	24137569	17	6.6115	368	135424	49836032	19.1833	7.1661
290	84100	24389000	17.0294	6.6191	369	136161	50243409	19.2094	7.1726
291	84681	24642171	17.0587	6.6267	370	136900	50653000	19.2354	7.1791
292	85264	24897088	17.0880	6.6343	371	137641	51064811	19.2614	7.1855
293	85849	25153757	17.1172	6.6419	372	138384	51478848	19.2873	7.1920
294	86436	25412184	17.1464	6.6494	373	139129	51895117	19.3132	7.1984
295	87025	25672375	17.1756	6.6569	374	139876	52313624	19.3391	7.2048
296	87616	25934336	17.2047	6.6644	375	140625	52734375	19.3649	7.2112
297	88209	26198073	17.2337	6.6719	376	141376	53157376	19.3907	7.2177
298	88804	26463592	17.2627	6.6794	377	142129	53582633	19.4165	7.2240
299	89401	26730899	17.2916	6.6869	378	142884	54010152	19.4422	7.2304
300	90000	27000000	17.3205	6.6943	379	143641	54439939	19.4679	7.2368
301	90601	27270901	17.3494	6.7018	380	144400	54872000	19.4936	7.2432
302	91204	27543608	17.3781	6.7092	381	145161	55306341	19.5192	7.2495
303	91809	27818127	17.4069	6.7166	382	145924	55742968	19.5448	7.2558
304	92416	28094464	17.4356	6.7240	383	146689	56181887	19.5704	7.2622
305	93025	28372625	17.4642	6.7313	384	147456	56623104	19.5959	7.2685
306	93636	28652616	17.4929	6.7387	385	148225	57066625	19.6214	7.2748
307	94249	28934443	17.5214	6.7460	386	148996	57512456	19.6469	7.2811
308	94864	29218112	17.5499	6.7533	387	149769	57960603	19.6723	7.2874
309	95481	29503629	17.5784	6.7606	388	150544	58411072	19.6977	7.2936
310	96100	29791000	17.6068	6.7679	389	151321	58863869	19.7231	7.2999
311	96721	30080231	17.6352	6.7752	390	152100	59319000	19.7484	7.3061
312	97344	30371328	17.6635	6.7824	391	152881	59776471	19.7737	7.3124
313	97969	30664297	17.6918	6.7897	392	153664	60236288	19.7990	7.3186
314	98596	30959144	17.7200	6.7969	393	154449	60698457	19.8242	7.3248
315	99225	31255875	17.7482	6.8041	394	155236	61162984	19.8494	7.3310
316	99856	31554496	17.7764	6.8113	395	156025	61629875	19.8746	7.3372
317	100489	31855013	17.8045	6.8185	396	156816	62099136	19.8997	7.3434
318	101124	32157432	17.8326	6.8256	397	157609	62570773	19.9249	7.3496
319	101761	32461759	17.8606	6.8328	398	158404	63044792	19.9499	7.3558
320	102400	32768000	17.8885	6.8399	399	159201	63521199	19.9750	7.3619
321	103041	33076161	17.9165	6.8470	400	160000	64000000	20.	7.3681
322	103684	33386248	17.9444	6.8541	401	160801	64481201	20.0250	7.3742
323	104329	33698267	17.9722	6.8612	402	161604	64964808	20.0499	7.3803
324	104976	34012224	18.	6.8683	403	162409	65450827	20.0749	7.3864
325	105625	34328125	18.0278	6.8753	404	163216	65939264	20.0998	7.3925
326	106276	34645976	18.0555	6.8824	405	164025	66430125	20.1246	7.3986
327	106929	34965783	18.0831	6.8894	406	164836	66923416	20.1494	7.4047
328	107584	35287552	18.1108	6.8964	407	165649	67419143	20.1742	7.4108
329	108241	35611289	18.1384	6.9034	408	166464	67917312	20.1990	7.4169
330	108900	35937000	18.1659	6.9104	409	167281	68417929	20.2237	7.4229
331	109561	36264691	18.1934	6.9174	410	168100	68921000	20.2485	7.4290
332	110224	36594368	18.2209	6.9244	411	168921	69426531	20.2731	7.4350
333	110889	36926037	18.2483	6.9313	412	169744	69934528	20.2978	7.4410
334	111556	37259704	18.2757	6.9382	413	170569	70444997	20.3224	7.4470
335	112225	37595375	18.3030	6.9451	414	171396	70957944	20.3470	7.4530
336	112896	37933056	18.3303	6.9521	415	172225	71473375	20.3715	7.4590
337	113569	38272753	18.3576	6.9589	416	173056	71991296	20.3961	7.4650
338	114244	38614472	18.3848	6.9658	417	173889	72511713	20.4206	7.4710
339	114921	38958219	18.4120	6.9727	418	174724	73034632	20.4450	7.4770
340	115600	39304000	18.4391	6.9795	419	175561	73560059	20.4695	7.4829
341	116281	39651821	18.4662	6.9864	420	176400	74088000	20.4939	7.4889
342	116964	40001688	18.4932	6.9932	421	177241	74618461	20.5183	7.4948
343	117649	40353607	18.5203	7.	422	178084	75151448	20.5426	7.5007
344	118336	40707584	18.5472	7.0068	423	178929	75686967	20.5670	7.5067
345	119025	41063625	18.5742	7.0136	424	179776	76225024	20.5913	7.5126
346	119716	41421736	18.6011	7.0203	425	180625	76765625	20.6155	7.5185
347	120409	41781923	18.6279	7.0271	426	181476	77308776	20.6398	7.5244
348	121104	42144192	18.6548	7.0338	427	182329	77854483	20.6640	7.5302
349	121801	42508549	18.6815	7.0406	428	183184	78402752	20.6882	7.5361

TABLE VI.—*Table of squares, cubes, square roots, and cube roots of numbers from 1 to 1,000*—Continued.

No.	Square.	Cube.	Square root.	Cube root.	No.	Square.	Cube.	Square root.	Cube root.
429	184041	78953589	20.7123	7.5420	588	345744	203297472	24.2487	8.3777
430	184900	79507000	20.7364	7.5478	509	259081	131872229	22.5610	7.9843
431	185761	80062991	20.7605	7.5537	510	260100	132651000	22.5832	7.9896
432	186624	80621568	20.7846	7.5595	511	261121	133432831	22.6053	7.9948
433	187489	81182737	20.8087	7.5654	512	262144	134217728	22.6274	8.
434	188356	81746504	20.8327	7.5712	513	263169	135005697	22.6495	8.0052
435	189225	82312875	20.8567	7.5770	514	264196	135796744	22.6716	8.0104
436	190096	82881856	20.8806	7.5828	515	265225	136590875	22.6936	8.0156
437	190969	83453453	20.9045	7.5886	516	266256	137388096	22.7156	8.0208
438	191844	84027672	20.9284	7.5944	517	267289	138188413	22.7376	8.0260
439	192721	84604519	20.9523	7.6001	518	268324	138991832	22.7596	8.0311
440	193600	85184000	20.9762	7.6059	519	269361	139798359	22.7816	8.0363
441	194481	85766121	21.	7.6117	520	270400	140608000	22.8035	8.0415
442	195364	86350888	21.0238	7.6174	521	271441	141420761	22.8254	8.0466
443	196249	86938307	21.0476	7.6232	522	272484	142236648	22.8473	8.0517
444	197136	87528384	21.0713	7.6289	523	273529	143055667	22.8692	8.0569
445	198025	88121125	21.0950	7.6346	524	274576	143877824	22.8910	8.0620
446	198916	88716536	21.1187	7.6403	525	275625	144703125	22.9129	8.0671
447	199809	89314623	21.1424	7.6460	526	276676	145531576	22.9347	8.0723
448	200704	89915392	21.1660	7.6517	527	277729	146363183	22.9565	8.0774
449	201601	90518849	21.1896	7.6574	528	278784	147197952	22.9783	8.0825
450	202500	91125000	21.2132	7.6631	529	279841	148035889	23.	8.0876
451	203401	91733851	21.2368	7.6688	530	280900	148877000	23.0217	8.0927
452	204304	92345408	21.2603	7.6744	531	281961	149721291	23.0434	8.0978
453	205209	92959677	21.2838	7.6801	532	283024	150568768	23.0651	8.1028
454	206116	93576664	21.3073	7.6857	533	284089	151419437	23.0868	8.1079
455	207025	94196375	21.3307	7.6914	534	285156	152273304	23.1084	8.1130
456	207936	94818816	21.3542	7.6976	535	286225	153130375	23.1301	8.1180
457	208849	95443993	21.3776	7.7026	536	287296	153990656	23.1517	8.1231
458	209764	96071912	21.4009	7.7082	537	288369	154854153	23.1733	8.1281
459	210681	96702579	21.4243	7.7138	538	289444	155720872	23.1948	8.1332
460	211600	97336000	21.4476	7.7194	539	290521	156590819	23.2164	8.1382
461	212521	97972181	21.4709	7.7250	540	291600	157464000	23.2379	8.1433
462	213444	98611128	21.4942	7.7306	541	292681	158340421	23.2594	8.1483
463	214369	99252847	21.5174	7.7362	542	293764	159220088	23.2809	8.1533
464	215296	99897344	21.5407	7.7418	543	294849	160103007	23.3024	8.1583
465	216225	100544625	21.5639	7.7473	544	295936	160989184	23.3238	8.1633
466	217156	101194696	21.5870	7.7529	545	297025	161878625	23.3452	8.1683
467	218089	101847563	21.6102	7.7584	546	298116	162771336	23.3666	8.1733
468	219024	102503232	21.6333	7.7639	547	299209	163667323	23.3880	8.1783
469	219961	103161709	21.6564	7.7695	548	300304	164566592	23.4094	8.1833
470	220900	103823000	21.6795	7.7750	549	301401	165469149	23.4307	8.1882
471	221841	104487111	21.7025	7.7805	550	302500	166375000	23.4521	8.1932
472	222784	105154048	21.7256	7.7860	551	303601	167284151	23.4734	8.1982
473	223729	105823817	21.7486	7.7915	552	304704	168196608	23.4947	8.2031
474	224676	106496424	21.7715	7.7970	553	305809	169112377	23.5160	8.2081
475	225625	107171875	21.7945	7.8025	554	306916	170031464	23.5372	8.2130
476	226576	107850176	21.8174	7.8079	555	308025	170953875	23.5584	8.2180
477	227529	108531333	21.8403	7.8134	556	309136	171879616	23.5797	8.2229
478	228484	109215352	21.8632	7.8188	557	310249	172808693	23.6008	8.2278
479	229441	109902239	21.8861	7.8243	558	311364	173741112	23.6220	8.2327
480	230400	110592000	21.9089	7.8297	559	312481	174676879	23.6432	8.2377
481	231361	111284641	21.9317	7.8352	560	313600	175616000	23.6643	8.2426
482	232324	111980168	21.9545	7.8406	561	314721	176558481	23.6854	8.2475
483	233289	112678587	21.9773	7.8460	562	315844	177504328	23.7065	8.2524
484	234256	113379904	22.	7.8514	563	316969	178453547	23.7276	8.2573
485	235225	114084125	22.0227	7.8568	564	318096	179406144	23.7487	8.2621
486	236196	114791256	22.0454	7.8622	565	319225	180362125	23.7697	8.2670
487	237169	115501303	22.0681	7.8676	566	320356	181321406	23.7908	8.2719
488	238144	116214272	22.0907	7.8730	567	321489	182284263	23.8118	8.2768
489	239121	116930169	22.1133	7.8784	568	322624	183250432	23.8328	8.2816
490	240100	117649000	22.1359	7.8837	569	323761	184220009	23.8537	8.2865
491	241081	118370771	22.1585	7.8891	570	324900	185193000	23.8747	8.2913
492	242064	119095488	22.1811	7.8944	571	326041	186169411	23.8956	8.2962
493	243049	119823157	22.2036	7.8998	572	327184	187149248	23.9165	8.3010
494	244036	120553784	22.2261	7.9051	573	328329	188132517	23.9374	8.3059
495	245025	121287375	22.2486	7.9105	574	329476	189119224	23.9583	8.3107
496	246016	122023936	22.2711	7.9158	575	330625	190109375	23.9792	8.3155
497	247009	122763473	22.2935	7.9211	576	331776	191102976	24.	8.3203
498	248004	123505992	22.3159	7.9264	577	332929	192100033	24.0208	8.3251
499	249001	124251499	22.3383	7.9317	578	334084	193100552	24.0416	8.3300
500	250000	125000000	22.3607	7.9370	579	335241	194104539	24.0624	8.3348
501	251001	125751501	22.3830	7.9423	580	336400	195112000	24.0832	8.3396
502	252004	126506008	22.4054	7.9476	581	337561	196122941	24.1039	8.3443
503	253009	127263527	22.4277	7.9528	582	338724	197137368	24.1247	8.3491
504	254016	128024064	22.4499	7.9581	583	339889	198155287	24.1454	8.3539
505	255025	128787625	22.4722	7.9634	584	341056	199176704	24.1661	8.3587
506	256036	129554216	22.4944	7.9686	585	342225	200201625	24.1868	8.3634
507	257049	130323843	22.5167	7.9739	586	343396	201230056	24.2074	8.3682
508	258064	131096512	22.5389	7.9791	587	344569	202262003	24.2281	8.3730

TABLE VI.—*Table of squares, cubes, square roots, and cube roots of numbers from 1 to 1,000*—Continued.

No.	Square.	Cube.	Square root.	Cube root.	No.	Square.	Cube.	Square root.	Cube root.
589	346921	204336469	24.2693	8.3825	668	446224	298077632	25.8457	8.7416
590	348100	205379000	24.2899	8.3872	669	447561	299418309	25.8650	8.7460
591	349281	206425071	24.3105	8.3919	670	448900	300763000	25.8844	8.7503
592	350464	207474688	24.3311	8.3967	671	450241	302111711	25.9037	8.7547
593	351649	208527857	24.3516	8.4014	672	451584	303464448	25.9230	8.7590
594	352836	209584584	24.3721	8.4061	673	452929	304821217	25.9422	8.7634
595	354025	210644875	24.3926	8.4108	674	454276	306182024	25.9615	8.7677
596	355216	211708736	24.4131	8.4155	675	455625	307546875	25.9808	8.7721
597	356409	212776173	24.4336	8.4202	676	456976	308915776	26.	8.7764
598	357604	213847192	24.4540	8.4249	677	458329	310288733	26.0192	8.7807
599	358801	214921799	24.4745	8.4296	678	459684	311665752	26.0384	8.7850
600	360000	216000000	24.4949	8.4343	679	461041	313046839	26.0576	8.7893
601	361201	217081801	24.5153	8.4390	680	462400	314432000	26.0768	8.7937
602	362404	218167208	24.5357	8.4437	681	463761	315821241	26.0960	8.7980
603	363609	219256227	24.5561	8.4484	682	465124	317214568	26.1151	8.8023
604	364816	220348864	24.5764	8.4530	683	466489	318611987	26.1343	8.8066
605	366025	221445125	24.5967	8.4577	684	467856	320013504	26.1534	8.8109
606	367236	222545016	24.6171	8.4623	685	469225	321419125	26.1725	8.8152
607	368449	223648543	24.6374	8.4670	686	470596	322828856	26.1916	8.8194
608	369664	224755712	24.6577	8.4716	687	471969	324242703	26.2107	8.8237
609	370881	225866529	24.6779	8.4763	688	473344	325660672	26.2298	8.8280
610	372100	226981000	24.6982	8.4809	689	474721	327082769	26.2488	8.8323
611	373321	228099131	24.7184	8.4856	690	476100	328509000	26.2679	8.8366
612	374544	229220928	24.7386	8.4902	691	477481	329939371	26.2869	8.8408
613	375769	230346397	24.7588	8.4948	692	478864	331373888	26.3059	8.8451
614	376996	231475544	24.7790	8.4994	693	480249	332812557	26.3249	8.8493
615	378225	232608375	24.7992	8.5040	694	481636	334255384	26.3439	8.8536
616	379456	233744896	24.8193	8.5086	695	483025	335702375	26.3629	8.8578
617	380689	234885113	24.8395	8.5132	696	484416	337153536	26.3818	8.8621
618	381924	236029032	24.8596	8.5178	697	485809	338608873	26.4008	8.8663
619	383161	237176659	24.8797	8.5224	698	487204	340068392	26.4197	8.8706
620	384400	238328000	24.8998	8.5270	699	488601	341532099	26.4386	8.8748
621	385641	239483061	24.9199	8.5316	700	490000	343000000	26.4575	8.8790
622	386884	240641848	24.9399	8.5362	701	491401	344472101	26.4764	8.8833
623	388129	241804367	24.9600	8.5408	702	492804	345948408	26.4953	8.8875
624	389376	242970624	24.9800	8.5453	703	494209	347428927	26.5141	8.8917
625	390625	244140625	25.	8.5499	704	495616	348913664	26.5330	8.8959
626	391876	245314376	25.0200	8.5544	705	497025	350402625	26.5518	8.9001
627	393129	246491883	25.0400	8.5590	706	498436	351895816	26.5707	8.9043
628	394384	247673152	25.0599	8.5635	707	499849	353393243	26.5895	8.9085
629	395641	248858180	25.0799	8.5681	708	501264	354894912	26.6083	8.9127
630	396900	250047000	25.0998	8.5726	709	502681	356400829	26.6271	8.9169
631	398161	251239591	25.1197	8.5772	710	504100	357911000	26.6458	8.9211
632	399424	252435968	25.1396	8.5817	711	505521	359425431	26.6646	8.9253
633	400689	253636137	25.1595	8.5862	712	506944	360944128	26.6833	8.9295
634	401956	254840104	25.1794	8.5907	713	508369	362467097	26.7021	8.9337
635	403225	256047875	25.1992	8.5952	714	509796	363994344	26.7208	8.9378
636	404496	257259456	25.2190	8.5997	715	511225	365525875	26.7395	8.9420
637	405769	258474853	25.2389	8.6043	716	512656	367061696	26.7582	8.9462
638	407044	259694072	25.2587	8.6088	717	514089	368601813	26.7769	8.9503
639	408321	260917119	25.2784	8.6132	718	515524	370146232	26.7955	8.9545
640	409600	262144000	25.2982	8.6177	719	516961	371694959	26.8142	8.9587
641	410881	263374721	25.3180	8.6222	720	518400	373248000	26.8328	8.9628
642	412164	264609288	25.3377	8.6267	721	519841	374805361	26.8514	8.9670
643	413449	265847707	25.3574	8.6312	722	521284	376367048	26.8701	8.9711
644	414736	267089984	25.3772	8.6357	723	522729	377933067	26.8887	8.9752
645	416025	268336125	25.3969	8.6401	724	524176	379503424	26.9072	8.9794
646	417316	269586136	25.4165	8.6446	725	525625	381078125	26.9258	8.9835
647	418609	270840023	25.4362	8.6490	726	527076	382657176	26.9444	8.9876
648	419904	272097792	25.4558	8.6535	727	528529	384240583	26.9629	8.9018
649	421201	273359449	25.4755	8.6579	728	529984	385828352	26.9815	8.9959
650	422500	274625000	25.4951	8.6624	729	531441	387420489	27.	9.
651	423801	275894451	25.5147	8.6668	730	532900	389017000	27.0185	9.0041
652	425104	277167808	25.5343	8.6713	731	534361	390617891	27.0370	9.0082
653	426409	278445077	25.5539	8.6757	732	535824	392223168	27.0555	9.0123
654	427716	279726264	25.5734	8.6801	733	537289	393832837	27.0740	9.0164
655	429025	281011375	25.5930	8.6845	734	538756	395446904	27.0924	9.0205
656	430336	282300416	25.6125	8.6890	735	540225	397065375	27.1109	9.0246
657	431649	283593393	25.6320	8.6934	736	541696	398688256	27.1293	9.0287
658	432964	284890312	25.6515	8.6978	737	543169	400315553	27.1477	9.0328
659	434281	286191179	25.6710	8.7022	738	544644	401947272	27.1662	9.0369
660	435600	287496000	25.6905	8.7066	739	546121	403583419	27.1846	9.0410
661	436921	288804781	25.7099	8.7110	740	547600	405224000	27.2029	9.0450
662	438244	290117528	25.7294	8.7154	741	549081	406869021	27.2213	9.0491
663	439569	291434247	25.7488	8.7198	742	550564	408518488	27.2397	9.0532
664	440896	292754944	25.7682	8.7241	743	552049	410172407	27.2580	9.0572
665	442225	294079625	25.7876	8.7285	744	553536	411830784	27.2764	9.0613
666	443556	295408296	25.8070	8.7329	745	555025	413493625	27.2947	9.0654
667	444889	296740963	25.8263	8.7373	746	556516	415160936	27.3130	9.0694

TABLE VI.—*Table of squares, cubes, square roots, and cube roots of numbers from 1 to 1,000*—Continued.

No.	Square.	Cube.	Square root.	Cube root.	No.	Square.	Cube.	Square root.	Cube root.
747	558009	416832723	27.3313	9.0735	826	682276	563559976	28.7402	9.3827
748	559504	418508992	27.3496	9.0775	827	683929	565609283	28.7576	9.3865
749	561001	420189749	27.3679	9.0816	828	685584	567663552	28.7750	9.3902
750	562500	421875000	27.3861	9.0856	829	687241	569722789	28.7924	9.3940
751	564001	423564751	27.4044	9.0896	830	688900	571787000	28.8097	9.3978
752	565504	425259008	27.4226	9.0937	831	690561	573856191	28.8271	9.4016
753	567009	426957777	27.4408	9.0977	832	692224	575930368	28.8444	9.4053
754	568516	428661064	27.4591	9.1017	833	693889	578009537	28.8617	9.4091
755	570025	430368875	27.4773	9.1057	834	695556	580093704	28.8791	9.4129
756	571536	432081216	27.4955	9.1098	835	697225	582182875	28.8964	9.4166
757	573049	433798093	27.5136	9.1138	836	698896	584277056	28.9137	9.4204
758	574564	435519512	27.5318	9.1178	837	700569	586376253	28.9310	9.4241
759	576081	437245479	27.5500	9.1218	838	702244	588480472	28.9482	9.4279
760	577600	438976000	27.5681	9.1258	839	703921	590589719	28.9655	9.4316
761	579121	440711081	27.5862	9.1298	840	705600	592704000	28.9828	9.4354
762	580644	442450728	27.6043	9.1338	841	707281	594823321	29.	9.4391
763	582169	444194947	27.6225	9.1378	842	708964	596947688	29.0172	9.4429
764	583696	445943744	27.6405	9.1418	843	710649	599077107	29.0345	9.4466
765	585225	447697125	27.6586	9.1458	844	712336	601211584	29.0517	9.4503
766	586756	449455096	27.6767	9.1498	845	714025	603351125	29.0689	9.4541
767	588289	451217663	27.6948	9.1537	846	715716	605495736	29.0861	9.4578
768	589824	452984832	27.7128	9.1577	847	717409	607645423	29.1033	9.4615
769	591361	454756609	27.7308	9.1617	848	719104	609800192	29.1204	9.4652
770	592900	456533000	27.7489	9.1657	849	720801	611960049	29.1376	9.4690
771	594441	458314011	27.7669	9.1696	850	722500	614125000	29.1548	9.4727
772	595984	460099648	27.7849	9.1736	851	724201	616295051	29.1719	9.4764
773	597529	461889917	27.8029	9.1775	852	725904	618470208	29.1890	9.4801
774	599076	463684824	27.8209	9.1815	853	727609	620650477	29.2062	9.4838
775	600625	465484375	27.8388	9.1855	854	729316	622835864	29.2233	9.4875
776	602176	467288576	27.8568	9.1894	855	731025	625026375	29.2404	9.4912
777	603729	469097433	27.8747	9.1933	856	732736	627222016	29.2575	9.4949
778	605284	470910952	27.8927	9.1973	857	724449	629422793	29.2746	9.4986
779	606841	472729139	27.9106	9.2012	858	736164	631628712	29.2916	9.5023
780	608400	474552000	27.9285	9.2052	859	737881	633839779	29.3087	9.5060
781	609961	476379541	27.9464	9.2091	860	739600	636056000	29.3258	9.5097
782	611524	478211768	27.9643	9.2130	861	741321	638277381	29.3428	9.5134
783	613089	480048687	27.9821	9.2170	862	743044	640503928	29.3598	9.5171
784	614656	481890304	28.	9.2209	863	744769	642735647	29.3769	9.5207
785	616225	483736625	28.0179	9.2248	864	746496	644972544	29.3939	9.5244
786	617796	485587656	28.0357	9.2287	865	748225	647214625	29.4109	9.5281
787	619369	487443403	28.0535	9.2326	866	749956	649461896	29.4279	9.5317
788	620944	489303872	28.0713	9.2365	867	751689	651714363	29.4449	9.5354
789	622521	491169069	28.0891	9.2404	868	753424	653972032	29.4618	9.5391
790	624100	493039000	28.1069	9.2443	869	755161	656234909	29.4788	9.5427
791	625681	494913671	28.1247	9.2482	870	756900	658503000	29.4958	9.5464
792	627264	496793088	28.1425	9.2521	871	758641	660776311	29.5127	9.5501
793	628849	498677257	28.1603	9.2560	872	760384	663054848	29.5296	9.5537
794	630436	500566184	28.1780	9.2599	873	762129	665338617	29.5466	9.5574
795	632025	502459875	28.1957	9.2638	874	763876	667627624	29.5635	9.5610
796	633616	504358336	28.2135	9.2677	875	765625	669921875	29.5804	9.5647
797	635209	506261573	28.2312	9.2716	876	767376	672221376	29.5973	9.5683
798	636804	508169592	28.2489	9.2754	877	769129	674526133	29.6142	9.5719
799	638401	510082399	28.2666	9.2793	878	770884	676836152	29.6311	9.5756
800	640000	512000000	28.2843	9.2832	879	772641	679151439	29.6479	9.5792
801	641601	513922401	28.3019	9.2870	880	774400	681472000	29.6648	9.5828
802	643204	515849608	28.3196	9.2909	881	776161	683797841	29.6816	9.5865
803	644809	517781627	28.3373	9.2948	882	777924	686128968	29.6985	9.5901
804	646416	519718464	28.3549	9.2986	883	779689	688465387	29.7153	9.5937
805	648025	521660125	28.3725	9.3025	884	781456	690807104	29.7321	9.5973
806	649636	523606616	28.3901	9.3063	885	783225	693154125	29.7489	9.6010
807	651249	525557943	28.4077	9.3102	886	784996	695506456	29.7658	9.6046
808	652864	527514112	28.4253	9.3140	887	786769	697864103	29.7825	9.6082
809	654481	529475129	28.4429	9.3179	888	788544	700227072	29.7993	9.6118
810	656100	531441000	28.4605	9.3217	889	790321	702595369	29.8161	9.6154
811	657721	533411731	28.4781	9.3255	890	792100	704969000	29.8329	9.6190
812	659344	535387328	28.4956	9.3294	891	793881	707347971	29.8496	9.6226
813	660969	537367797	28.5132	9.3332	892	795664	709732288	29.8664	9.6262
814	662596	539353144	28.5307	9.3370	893	797449	712121957	29.8831	9.6298
815	664225	541343375	28.5482	9.3408	894	799236	714516984	29.8998	9.6334
816	665856	543338496	28.5657	9.3447	895	801025	716917375	29.9166	9.6370
817	667489	545338513	28.5832	9.3485	896	802816	719323136	29.9333	9.6406
818	669124	547343432	28.6007	9.3523	897	804609	721734273	29.9500	9.6442
819	670761	549353259	28.6182	9.3561	898	806404	724150792	29.9666	9.6477
820	672400	551368000	28.6356	9.3599	899	808201	726572699	29.9833	9.6513
821	674041	553387661	28.6531	9.3637	900	810000	729000000	30.	9.6549
822	675684	555412248	28.6705	9.3675	901	811801	731432701	30.0167	9.6585
823	677329	557441767	28.6880	9.3713	902	813604	738870808	30.0333	9.6620
824	678976	559476224	28.7054	9.3751	903	815409	736314327	30.0500	9.6656
825	680625	561515625	28.7228	9.3789	904	817216	738763264	30.0666	9.6692

TABLE VI.—*Table of squares, cubes, square roots, and cube roots of numbers from 1 to 1,000*—Continued.

No.	Square.	Cube.	Square root.	Cube root.	No.	Square.	Cube.	Square root.	Cube root.
905..........	819025	741217625	30.0832	9.6727	953..........	908209	865523177	30.8707	9.8408
906..........	820836	743677416	30.0998	9.6763	954..........	910116	868250664	30.8869	9.8443
907..........	822649	746142643	30.1164	9.6799	955..........	912025	870983875	30.9031	9.8477
908..........	824464	748613312	30.1330	9.6834	956..........	913936	873722816	30.9192	9.8511
909..........	826281	751089429	30.1496	9.6870	937..........	915849	876467493	30.9354	9.8546
910..........	828100	753571000	30.1662	9.6905	958..........	917764	879217912	30.9516	9.8580
911..........	829921	756058031	30.1828	9.6941	959..........	919681	881974079	30.9677	9.8614
912..........	831744	758550528	30.1993	9.6976	960..........	921600	884736000	30.9839	9.8648
913..........	833569	761048497	30.2159	9.7012	961..........	923521	887503681	31.	9.8683
914..........	835396	763551944	30.2324	9.7047	962..........	925444	890277128	31.0161	9.8717
915..........	837225	766060875	30.2490	9.7082	963..........	927369	893056347	31.0322	9.8751
916..........	839056	768575296	30.2655	9.7118	964..........	929296	895841344	31.0483	9.8785
917..........	840889	771095213	30.2820	9.7153	965..........	931225	898632125	31.0644	9.8819
918..........	842724	773620632	30.2985	9.7188	966..........	933156	901428696	31.0805	9.8854
919..........	844561	776151559	30.3150	9.7224	967..........	935089	904231063	31.0966	9.8888
920..........	846400	778688000	30.3315	9.7259	968..........	937024	907039232	31.1127	9.8922
921..........	848241	781229961	30.3480	9.7294	969..........	938961	909853209	31.1288	9.8956
922..........	850084	783777448	30.3645	9.7329	970..........	940900	912673000	31.1448	9.8990
973..........	851929	786330467	30.3809	9.7364	971..........	942841	915498611	31.1609	9.9024
924..........	853776	788889024	30.3974	9.7400	972..........	944784	918330048	31.1769	9.9058
925..........	855625	791453125	30.4138	9.7435	973..........	946729	921167317	31.1929	9.9092
926..........	857476	794022776	30.4302	9.7470	974..........	948676	924010424	31.2090	9.9126
927..........	859329	796597983	30.4467	9.7505	975..........	950625	926859375	31.2250	9.9160
928..........	861184	799178752	30.4631	9.7540	976..........	952576	929714176	31.2410	9.9194
929..........	863041	801765089	30.4795	9.7575	977..........	954529	932574833	31.2570	9.9227
930..........	864900	804357000	30.4959	9.7610	978..........	956484	935441352	31.2730	9.9261
931..........	866761	806954491	30.5123	9.7645	979..........	958441	938313739	31.2890	9.9295
932..........	868624	809557568	30.5287	9.7680	980..........	960400	941192000	31.3050	9.9329
933..........	870489	812166237	30.5450	9.7715	981..........	962361	944076141	31.3209	9.9363
934..........	872356	814780504	30.5614	9.7750	982..........	964324	946966168	31.3369	9.9396
935..........	874225	817400375	30.5778	9.7785	983..........	966289	949862087	31.3528	9.9430
936..........	876096	820025856	30.5941	9.7819	984..........	968256	952763904	31.3688	9.9464
937..........	877969	822656953	30.6105	9.7854	985..........	970225	955671625	31.3847	9.9497
938..........	879844	825293672	30.6268	9.7889	986..........	972196	958585256	31.4006	9.9531
939..........	881721	827936019	30.6431	9.7924	987..........	974169	961504803	31.4166	9.9565
940..........	883600	830584000	30.6594	9.7959	988..........	976144	964430272	31.4325	9.9598
941..........	885481	833237621	30.6757	9.7993	989..........	978121	967361669	31.4484	9.9632
942..........	887364	835896888	30.6920	9.8028	990..........	980100	970029000	31.4643	9.9666
943..........	889249	838561807	30.7083	9.8063	991..........	982081	973242271	31.4802	9.9699
944..........	891136	841232384	30.7246	9.8097	992..........	984064	976191488	31.4960	9.9733
945..........	893025	843908625	30.7409	9.8132	993..........	986049	979146657	31.5119	9.9766
946..........	894916	846590536	30.7571	9.8167	994..........	988036	982107784	31.5278	9.9800
947..........	896809	849278123	30.7734	9.8201	995..........	990025	985074875	31.5436	9.9833
948..........	898704	851971392	30.7896	9.8236	996..........	992016	988047936	31.5595	9.9866
949..........	900601	854670349	30.8058	9.8270	997..........	994009	991026973	31.5753	9.9900
950..........	902500	857375000	30.8221	9.8305	998..........	996004	994011992	31.5911	9.9933
951..........	904401	860085351	30.8383	9.8339	999..........	998001	997002999	31.6070	9.9967
952..........	906304	862801408	30.8545	9.8374	1000..........	1000000	100000000	31.6228	10.

213. To find the **square root of a decimal fraction or mixed number** from the foregoing table, multiply by 100 or by 10,000 and find the product in the column of squares. The corresponding number in the first column, with the decimal point one or two places to the left is the desired root.

For the cube root of a similar number, multiply by 1,000 or by 1,000,000, and find the nearest number in column of cubes. The corresponding number in the first column, with the decimal point one or two places to the left, is the desired root.

Examples: Required the square root of 5.246.

Multiply by 100; the result is 524, which found in column of squares is opposite 23 in the column of numbers. Moving the decimal point one place to the left to correspond with the multiplication by 100, gives 2.3 for the desired square root, to the first place of decimals and hence approximate only. Second: Multiply by 10,000; the result is 52,460, which found in the column of squares is opposite

229 in the column of numbers. Moving the decimal point two places to the left to correspond to the multiplication by 10,000, the result is 2.29, which is the desired root to the second place of decimals.

Required the cube root of 5.246. Multiply by 1,000, giving 5,246, which found in the column of cubes is opposite the number 17 in the first column. Moving the decimal point one place to the left to correspond to the multiplication by 1,000 gives 1.7, which is the required cube root to one decimal place. Again, multiplying by 1,000,000 gives 5,246,000, which found in the column of cubes is opposite the number 174 in the column of numbers. Moving the decimal point two places to the left to correspond with the multiplication by 1,000,000, gives the number 1.74, which is the desired cube root correct to two places of decimals.

To **find the square root or cube root of a number greater** than 1,000, find the nearest number in the column of squares or cubes and take the corresponding number in the first column, which will be correct for the number of figures it contains.

For the **fourth root**, take the square root of the square root. For the **sixth root**, the square root of the cube root, or the cube root of the square root. **Higher roots**, the indices of which can be factored in 3's and 2's, may be taken in the same way.

CIRCULAR FUNCTIONS.

214. Those most used are shown graphically in fig. 68. They bear a definite relation to the radius of a circle in which they are drawn. When the radius is unity, functions are called **natural**, as natural sine, natural tangent, etc. Their values are given in Table XVI for each 10′ of arc. The tabulated values are ratios of the several functions to the radius and if any length, expressed in any unit, considered as a radius, be multiplied by a tabular number, the result will be the corresponding funetion of the circle of the given radius. The table gives values from 0 to 90°. For greater angles, use the following relations: Subtract the given angle from 180° or 360°, or subtract 180° from the angle, as may be required, to leave a remainder of 90° or less. Take out the required function of the remainder, which is also that of the given angle.

Interpolation for values not in the table may be done approximately by taking the proportional amount of the difference between two consecutive values. Thus, for the sine of 28° 43′ take the sine of 28° 40′ plus $\frac{3}{10}$ of the difference between sine 28° 40′ and sine 28° 50′.

TABLE VII.—*Natural sines and tangents to a radius 1.*

Arc.		Sine.	Tang.	Cotang.	Cosine.		
°	′					°	′
0	00	.0000000	.000000	Infinite.	1.0000000	90	00
	10	.0029089	.002909	343.7737	.9999958		50
	20	.0058177	.005817	171.8854	.9999831		40
	30	.0087265	.008726	114.5886	.9999619		30
	40	.0116353	.011636	85.93979	.9999323		20
	50	.0145439	.014545	68.75008	.9998942		10
1	00	.0174524	.017455	57.28996	.9998477	89	00
	10	.0203608	.020365	49.10388	.9997927		50
	20	.0232690	.023275	42.96407	.9997292		40
	30	.0261769	.026185	38.18845	.9996573		30
	40	.0290847	.029097	34.36777	.9995770		20
	50	.0319922	.032008	31.24157	.9994881		10
2	00	.0348995	.034920	28.63625	.9993908	88	00
	10	.0378065	.037833	26.43160	.9992851		50
	20	.0407131	.040746	24.54175	.9991709		40
	30	.0436194	.043660	22.90376	.9990482		30
	40	.0465253	.046575	21.47040	.9989171		20
	50	.0494308	.049491	20.20555	.9987775		10
3	00	.0523360	.052407	19.08113	.9986295	87	00
	10	.0552406	.055325	18.07497	.9984731		50
	20	.0581448	.058243	17.16933	.9983082		40
	30	.0610485	.061162	16.34985	.9981348		30
	40	.0639517	.064082	15.60478	.9979530		20
	50	.0668544	.067004	14.92441	.9977627		10
4	00	.0697565	.069926	14.30066	.9975641	86	00
	10	.0726580	.072850	13.72673	.9973569		50
	20	.0755589	.075775	13.19688	.9971413		40
	30	.0784591	.078701	12.70620	.9969173		30
	40	.0813587	.081629	12.25050	.9966849		20
	50	.0842576	.084558	11.82616	.9964440		10
5	00	.0871557	.087488	11.43005	.9961947	85	00
	10	.0900532	.090420	11.05943	.9959370		50
	20	.0929499	.093354	10.71191	.9956708		40
	30	.0958458	.096289	10.38539	.9953962		30
	40	.0987408	.099225	10.07803	.9951132		20
	50	.1016351	.102164	9.788173	.9948217		10
6	00	.1045285	.105104	9.514364	.9945219	84	00
	10	.1074210	.108046	9.255303	.9942136		50
	20	.1103126	.110989	9.009826	.9938969		40
	30	.1132032	.113935	8.776887	.9935719		30
	40	.1160929	.116883	8.555546	.9932384		20
	50	.1189816	.119832	8.344955	.9928965		10
7	00	.1218693	.122784	8.144346	.9925462	83	00
	10	.1247560	.125738	7.953022	.9921874		50
	20	.1276416	.128694	7.770350	.9918204		40
	30	.1305262	.131652	7.595754	.9914449		30
	40	.1334096	.134612	7.428706	.9910610		20
	50	.1362919	.137575	7.268725	.9906687		10
		Cosine.	Cotang.	Tang.	Sine.	Arc.	

TABLE VII.—*Natural sines and tangents*—Continued.

Arc.		Sine.	Tang.	Cotang.	Cosine.		
°	′					°	′
8	00	.1391731	.140540	7.115369	.9902681	82	00
	10	.1420531	.143508	6.968233	.9898590		50
	20	.1449319	.146478	6.826943	.9894416		40
	30	.1478094	.149451	6.691156	.9890159		30
	40	.1506857	.152426	6.560553	.9885817		20
	50	.1535607	.155404	6.434842	.9881392		10
9	00	.1564345	.158384	6.313751	.9876883	81	00
	10	.1593069	.161367	6.197027	.9872291		50
	20	.1621779	.164353	6.084438	.9867615		40
	30	.1650476	.167342	5.975764	.9862856		30
	40	.1679159	.170334	5.870804	.9858013		20
	50	.1707828	.173329	5.769368	.9853087		10
10	00	.1736482	.176327	5.671281	.9848078	80	00
	10	.1765121	.179327	5.576378	.9842985		50
	20	.1793746	.182331	5.484505	.9837808		40
	30	.1822355	.185339	5.395517	.9832549		30
	40	.1850949	.188349	5.309279	.9827206		20
	50	.1879528	.191363	5.225664	.9821781		10
11	00	.1908090	.194380	5.144554	.9816272	79	00
	10	.1936636	.197400	5.065835	.9810680		50
	20	.1965166	.200424	4.989402	.9805005		40
	30	.1993679	.203452	4.915157	.9799247		30
	40	.2022176	.206483	4.843004	.9793406		20
	50	.2050655	.209518	4.772856	.9787483		10
12	00	.2079117	.212556	4.704630	.9781476	78	00
	10	.2107561	.215598	4.638245	.9775386		50
	20	.2135988	.218644	4.573628	.9769215		40
	30	.2164396	.221694	4.510708	.9762960		30
	40	.2192786	.224748	4.449418	.9756623		20
	50	.2221158	.227806	4.389694	.9750203		10
13	00	.2249511	.230868	4.331475	.9743701	77	00
	10	.2277844	.233934	4.274706	.9737116		50
	20	.2306159	.237004	4.219331	.9730449		40
	30	.2334454	.240078	4.165299	.9723699		30
	40	.2362729	.243157	4.112561	.9716867		20
	50	.2390984	.246240	4.061070	.9709953		10
14	00	.2419219	.249328	4.010780	.9702957	76	00
	10	.2447433	.252420	3.961651	.9695879		50
	20	.2475627	.255516	3.913642	.9688719		40
	30	.2503800	.258617	3.866713	.9681476		30
	40	.2531952	.261723	3.820828	.9674152		20
	50	.2560082	.264833	3.775951	.9666746		10
15	00	.2588190	.267949	3.732050	.9659258	75	00
	10	.2616277	.271069	3.689092	.9651689		50
	20	.2644342	.274194	3.647046	.9644037		40
	30	.2672384	.277324	3.605883	.9636305		30
	40	.2700403	.280459	3.565574	.9628490		20
	50	.2728400	.283599	3.526093	.9620594		10
		Cosine.	Cotang.	Tang.	Sine.	Arc.	

TABLE VII.—*Natural sines and tangents*—Continued.

Arc.		Sine.	Tang.	Cotang.	Cosine.		
°	′					°	′
16	00	.2756374	.268745	3.487414	.9612617	74	00
	10	.2784324	.289896	3.449512	.9604558		50
	20	.2812251	.293052	3.412362	.9596418		40
	30	.2840153	.296213	3.375943	.9588197		30
	40	.2868032	.299380	3.340232	.9579895		20
	50	.2895887	.302552	3.305209	.9571512		10
17	00	.2923717	.305730	3.270852	.9563048	73	00
	10	.2951522	.308914	3.237143	.9554502		50
	20	.2979303	.312103	3.204063	.9545876		40
	30	.3007058	.315298	3.171594	.9537170		30
	40	.3034788	.318499	3.139719	.9528382		20
	50	.3062492	.321706	3.108421	.9519514		10
18	00	.3090170	.324919	3.077683	.9510565	72	00
	10	.3117822	.328138	3.047491	.9501536		50
	20	.3145448	.331363	3.017830	.9492426		40
	30	.3173047	.334595	2.988685	.9483237		30
	40	.3200619	.337833	2.960042	.9473966		20
	50	.3228164	.341077	2.931888	.9464616		10
19	00	.3255682	.344327	2.904210	.9455186	71	00
	10	.3283172	.347584	2.876997	.9445675		50
	20	.3310634	.350848	2.850234	.9436085		40
	30	.3338069	.354118	2.823912	.9426415		30
	40	.3365475	.357395	2.798019	.9416665		20
	50	.3392852	.360679	2.772544	.9406835		10
20	00	.3420201	.363970	2.747477	.9396926	70	00
	10	.3447521	.367268	2.722807	.9386938		50
	20	.3474812	.370572	2.698525	.9376869		40
	30	.3502074	.373884	2.674621	.9366722		30
	40	.3529306	.377203	2.651086	.9356495		20
	50	.3556508	.380530	2.627912	.9346189		10
21	00	.3583679	.383864	2.605089	.9335804	69	00
	10	.3610821	.387205	2.582609	.9325340		50
	20	.3637932	.390554	2.560464	.9314797		40
	30	.3665012	.393910	2.538647	.9304176		30
	40	.3692061	.397274	2.517150	.9293475		20
	50	.3719079	.400646	2.495966	.9282696		10
22	00	.3746066	.404026	2.475086	.9271839	68	00
	10	.3773021	.407413	2.454506	.9260902		50
	20	.3799944	.410809	2.434217	.9249888		40
	30	.3826834	.414213	2.414213	.9238795		30
	40	.3853693	.417625	2.394488	.9227624		20
	50	.3880518	.421046	2.375037	.9216375		10
23	00	.3907311	.424474	2.355852	.9205049	67	00
	10	.3934071	.427912	2.336928	.9193644		50
	20	.3960798	.431357	2.318260	.9182161		40
	30	.3987491	.434812	2.299842	.9170601		30
	40	.4014150	.438275	2.281669	.9158963		20
	50	.4040775	.441747	2.263735	.9147247		10
		Cosine.	Cotang.	Tang.	Sine.	Arc.	

TABLE VII.—*Natural sines and tangents*—Continued.

Arc.		Sine.	Tang.	Cotang.	Cosine.		
°	′					°	′
24	00	.4067366	.445228	2.246036	.9135455	66	00
	10	.4093923	.448718	2.228567	.9123584		50
	20	.4120445	.452217	2.211323	.9111637		40
	30	.4146932	.455726	2.194299	.9099613		30
	40	.4173385	.459243	2.177492	.9087511		20
	50	.4199801	.462771	2.160895	.9075333		10
25	00	.4226183	.466307	2.144506	.9063078	65	00
	10	.4252528	.469853	2.128321	.9050746		50
	20	.4278838	.473409	2.112334	.9038338		40
	30	.4305111	.476975	2.096543	.9025853		30
	40	.4331348	.480551	2.080943	.9013292		20
	50	.4357548	.484136	2.065531	.9000654		10
26	00	.4383711	.487732	2.050303	.8987940	64	00
	10	.4409838	.491338	2.035256	.8975151		50
	20	.4435927	.494954	2.020386	.8962285		40
	30	.4461978	.498581	2.005689	.8949344		30
	40	.4487992	.502218	1.991163	.8936326		20
	50	.4513967	.505866	1.976805	.8923234		10
27	00	.4539905	.509525	1.962610	.8910065	63	00
	10	.4565804	.513195	1.948577	.8896822		50
	20	.4591665	.516875	1.934702	.8883503		40
	30	.4617486	.520567	1.920982	.8870108		30
	40	.4643269	.524269	1.907414	.8856639		20
	50	.4669012	.527983	1.893997	.8843095		10
28	00	.4694716	.531709	1.880726	.8829476	62	00
	10	.4720380	.535446	1.867600	.8815782		50
	20	.4746004	.539195	1.854615	.8802014		40
	30	.4771588	.542955	1.841770	.8788171		30
	40	.4797131	.546728	1.829062	.8774254		20
	50	.4822634	.550512	1.816489	.8760263		10
29	00	.4848096	.554309	1.804047	.8746197	61	00
	10	.4873517	.558117	1.791736	.8732058		50
	20	.4898897	.561939	1.779552	.8717844		40
	30	.4924236	.565772	1.767494	.8703557		30
	40	.4949532	.569619	1.755559	.8689196		20
	50	.4974787	.573478	1.743745	.8674762		10
30	00	.5000000	.577350	1.732050	.8660254	60	00
	10	.5025170	.581235	1.720473	.8645673		50
	20	.5050298	.585133	1.709011	.8631019		40
	30	.5075384	.589045	1.697663	.8616292		30
	40	.5100426	.592969	1.686426	.8601491		20
	50	.5125425	.596908	1.675298	.8586619		10
31	00	.5150381	.600860	1.664279	.8571673	59	00
	10	.5175293	.604826	1.653366	.8556655		50
	20	.5200161	.608806	1.642557	.8541504		40
	30	.5224986	.612800	1.631851	.8526402		30
	40	.5249766	.616809	1.621246	.8511167		20
	50	.5274502	.620832	1.610741	.8495860		10
		Cosine.	Cotang.	Tang.	Sine.	Arc.	

TABLE VII.—*Natural sines and tangents*—Continued.

Arc.		Sine.	Tang.	Cotang.	Cosine.		
°	′					°	′
32	00	.5299193	.624869	1.600334	.8480481	58	60
	10	.5323839	.628921	1.590023	.8465130		50
	20	.5348440	.632988	1.579807	.8449608		40
	30	.5372996	.637070	1.569685	.8433914		30
	40	.5397507	.641167	1.559655	.8418249		20
	50	.5421971	.645279	1.549715	.8402513		10
33	00	.5446390	.649407	1.539865	.8386706	57	60
	10	.5470763	.653551	1.530102	.8370827		50
	20	.5495090	.657710	1.520426	.8354878		40
	30	.5519370	.661885	1.510835	.8338858		30
	40	.5543603	.666076	1.501328	.8322768		20
	50	.5567790	.670284	1.491903	.8306607		10
34	00	.5591929	.674508	1.482561	.8290376	56	60
	10	.5616021	.678749	1.473298	.8274074		50
	20	.5640066	.683006	1.464114	.8257703		40
	30	.5664062	.687281	1.455009	.8241262		30
	40	.5688011	.691572	1.445980	.8224751		20
	50	.5711912	.695881	1.437026	.8208170		10
35	00	.5735764	.700207	1.428148	.8191520	55	60
	10	.5759568	.704551	1.419342	.8174801		50
	20	.5783323	.708913	1.410609	.8158013		40
	30	.5807030	.713293	1.401948	.8141155		30
	40	.5830687	.717691	1.393357	.8124229		20
	50	.5854294	.722107	1.384835	.8107234		10
36	00	.5877853	.726543	1.376381	.8090170	54	60
	10	.5901361	.730996	1.367995	.8073038		50
	20	.5924819	.735469	1.359676	.8055837		40
	30	.5948228	.739961	1.351422	.8038569		30
	40	.5971586	.744472	1.343233	.8021232		20
	50	.5994893	.749003	1.335107	.8003827		10
37	00	.6018150	.753554	1.327044	.7986355	53	60
	10	.6041356	.758124	1.319044	.7968815		50
	20	.6064511	.762715	1.311104	.7951208		40
	30	.6087614	.767327	1.303225	.7933533		30
	40	.6110666	.771958	1.295405	.7915792		20
	50	.6133666	.776611	1.287644	.7897983		10
38	00	.6156615	.781285	1.279941	.7880108	52	60
	10	.6179511	.785980	1.272295	.7862165		50
	20	.6202355	.790697	1.264706	.7844157		40
	30	.6225146	.795435	1.257172	.7826082		30
	40	.6247885	.800196	1.249693	.7807940		20
	50	.6270571	.804979	1.242268	.7789733		10
39	00	.6293204	.809784	1.234897	.7771460	51	60
	10	.6315784	.814611	1.227578	.7753121		50
	20	.6338310	.819462	1.220312	.7734716		40
	30	.6360782	.824336	1.213097	.7716246		30
	40	.6383201	.829233	1.205932	.7697710		20
	50	.6405566	.834154	1.198818	.7679110		10
		Cosine.	Cotang.	Tang.	Sine.	Arc.	

TABLE VII.—*Natural sines and tangents*—Continued.

Arc.		Sine.	Tang.	Cotang.	Cosine.		
°	′					°	′
40	00	.6427876	.839099	1.191753	.7660444	50	00
	10	.6450132	.844068	1.184737	.7641714		50
	20	.6472334	.849062	1.177769	.7622919		40
	30	.6494480	.854080	1.170849	.7604060		30
	40	.6516572	.859124	1.163976	.7585136		20
	50	.6538609	.864192	1.157149	.7566148		10
41	00	.6560590	.869286	1.150368	.7547096	49	00
	10	.6582516	.874406	1.143632	.7527980		50
	20	.6604386	.879552	1.136941	.7508800		40
	30	.6626200	.884725	1.130294	.7489557		30
	40	.6647959	.889924	1.123690	.7470251		20
	50	.6669661	.895150	1.117130	.7450881		10
42	00	.6691306	.900404	1.110612	.7431448	48	00
	10	.6712895	.905685	1.104136	.7411953		50
	20	.6734427	.910994	1.097702	.7392394		40
	30	.6755902	.916331	1.091308	.7372773		30
	40	.6777320	.921696	1.084955	.7353090		20
	50	.6798681	.927091	1.078642	.7333345		10
43	00	.6819984	.932515	1.072368	.7313537	47	00
	10	.6841229	.937968	1.066134	.7293668		50
	20	.6862416	.943451	1.059938	.7273736		40
	30	.6883546	.948964	1.053780	.7253744		30
	40	.6904617	.954508	1.047659	.7233690		20
	50	.6925630	.960082	1.041576	.7213574		10
44	00	.6946584	.965688	1.035530	.7193398	46	00
	10	.6967479	.971326	1.029520	.7173161		50
	20	.6988315	.976995	1.023546	.7152863		40
	30	.7009093	.982697	1.017607	.7132504		30
	40	.7029811	.988431	1.011703	.7112086		20
	50	.7050469	.994199	1.005834	.7091607		10
45	00	.7071068	1.000000	1.000000	.7071068	45	00
		Cosine.	Cotang.	Tang.	Sine.	Arc.	

PROPERTIES OF CIRCLES.

215. The ratio of the diameter to the circumference is represented in mathematics by π, called **Pi.** Its value can not be exactly expressed. To 5 decimal places it is 3.14159, which equals $\frac{22}{7}$ nearly. Log. π equals 0.4971499.

Diam. $\times 3.14159 =$ circ.

Diam. $\times 0.886277 =$ side of square of equal area.

Diam. $\times 0.7071 =$ side of inscribed square.

$\frac{1}{4}\pi D^2 = 0.7854 \times D^2 =$ area of the circle.

$\pi r^2 = 3.1416 \times r^2 =$ area of the circle.

The length of an arc of $n° = rn \times 0.017453$.

Example: If the radius is 542 feet, the length of an arc of 18° 20′ $= 18°.33 \times 542 \times 0.017453 = 165.5$ feet.

PROPERTIES OF SOME PLANE FIGURES.

216. Triangles are classed as equilateral when the three sides are of equal length; isoceles, when two sides only are equal; acute-angled, when each of its angles is less than 90°; obtuse-angled, when one angle is greater than 90°.

The sum of the angles of any triangle is 180°. The sides are directly proportional to the sines of the opposite angles, the greatest and least sides opposite the greatest and least angles.

Formulas for the solution of plane triangles. (Fig. 69.)

Given two sides, as a and b, and an angle opposite to one of them, as B.

$$\text{Sin. } A=\frac{a \sin. B}{b};\ C=180^\circ-(A+B);\ c=\frac{a \sin. C}{\sin. A}.$$

Given 2 angles, as A and B, and the included side c, the most common case.

$$C=180^\circ-(A+B);\ a=\frac{c \sin. A}{\sin. C};\ b=\frac{a \sin. B}{\sin. A}.$$

Given 2 sides at a and b, and the included angle C.

$$180^\circ-C=A+B$$

$$\text{Tan. } \frac{A-B}{2}=\frac{(a-b)\ \dfrac{(A+B)}{2}}{a+b}$$

$$A=\frac{A+B}{2}+\frac{A-B}{2};\ B=\frac{A+B}{2}-\frac{A-B}{2};\ C=\frac{a \sin. C}{\sin. A}.$$

Given the 3 sides—

$$\frac{a+b+c}{2}=S;\ \sin. \frac{A}{2}=\sqrt{\frac{(S-b)\ (S-c)}{bc}};$$

$$\sin. \frac{B}{2}=\sqrt{\frac{(S-a)\ (S-c)}{ac}};\ \sin. \frac{C}{2}=\sqrt{\frac{(S-a)\ (S-b)}{ab}}.$$

For every right-angled triangle the sine of the right angle is 1, and the following relations result: The side opposite the right angle is called the **hypothenuse.** (Fig. 69.)

$$\text{Hypothenuse}=a=c\div\sin. C=c\times\sec. B=\frac{b}{\text{Cos. } C}$$
$$=b\times\sec. C=\sqrt{b^2+c^2};$$
$$b=a\times\sin. B=a\times\cos. C=c\times\text{cotang. } C=c\times\text{tang. } B;$$
$$c=a\times\sin. C=a\times\cos. B=b\times\text{tang. } C;$$
$$\sin. B=\frac{b}{a}\cos. C;\ \sin. C=\frac{c}{a}=\cos. B;$$
$$\text{tang. } B=\frac{b}{c}=\text{cotang. } C;\ \text{tang. } C=\frac{c}{b}=\text{cotang. } B.$$

The area of a triangle equals any side multiplied by ½ the perpendicular distance from that side to the opposite angle. If the perpendicular from the angle does not intersect the opposite side, prolong the side, but do not include the prolongation in its length for computing the area. All triangles which have a common side and their opposite angles in a straight line parallel to the common side are equal in area.

A line bisecting one angle divides the opposite side into parts proportional to the adjacent sides. In fig. 70, *ab* bisects the angle at *a* and *bc*:*ac*::*bd*:*ad*.

Lines drawn from each angle to the middle of the opposite side intersect in a common point, which is the center of gravity of the triangle. The shorter part of each line is ½ the longer (fig. 71).

A line joining the middle points of two sides is parallel to the third side and ½ its length. In fig. 71 the line *ef*, joining the middle points of *ab* and *bc*, is parallel to *ac*, and ½ its length. Lines joining *eg* and *fg* would be parallel to *ab* and *bc*, and half their length, respectively.

Similar triangles are those which have the same angles and differ only in length of sides. The ratio between corresponding sides of all similar triangles is the same, since it is the ratio of the same function of the same angles. Hence, if two sides of a triangle and one of the corresponding sides of a similar triangle are known, the other corresponding side may be determined. The simplest test of similar triangles is that their corresponding sides are parallel or perpendicular. The principle of similar triangles is of great utility in field geometry.

The side of a square equals the diameter of an inscribed circle; or the diagram of a circumscribed circle × 0.7071. The diagonal of a square equals one side × 1.4142.

The area of a trapezoid, fig. 72, equals ½ the sum of the parallel sides *ab* and *cd* multiplied by the distance between them, *ef*.

The area of a trapezium—no two sides parallel—figure 73, equals ½ the diagonal *ac* multiplied by the sum of the perpendiculars, *bf* and *de*.

The side of a hexagon equals the radius of a circumscribed circle. The area equals the square of 1 side × 2.598. The side of an octagon equals the radius of a circumscribed circle × 0.7633. The area equals the square of one side × 4.8289.

To draw an octagon in a square (fig. 74).—From each corner, with a radius equal to ½ the diagonal, describe arcs as shown. Join the point at which they cut the sides. If a square stick be scribed at a distance from each corner equal to 0.3, the side of the square and the corners chamfered to the marks, the resulting section will be nearly a true octagon.

GEOMETRICAL CONSTRUCTIONS.

217. To divide a straight line into any number of equal parts: From one end of the line draw another, making any convenient angle with it, as 10° or 20°. On this auxiliary line lay off any assumed distance as many times as the number of equal parts desired. Join the last point so determined with the end of the first line. Through each of the points marked on the auxiliary line draw a line parallel to the line joining the ends. These lines will divide the given line into the desired number of equal parts.

To draw a perpendicular from a given point on a line: Mark 2 points equidistant from the given point, fig. 75, and with them as centers and a radius greater than their distance from the given point describe arcs on each side of the line. Connect one intersection with the given point by a straight line, which is the perpendicular required. As a check on accuracy, note whether the line passes through the other intersection.

If the given point is at one end of the line, from a convenient point *c* outside the line describe a semicircle passing through the given point and cutting the line again as at *b*, fig. 78. Draw a straight line *bc* through the center to the arc on the other side, as at *d*. The line *da* is the perpendicular required.

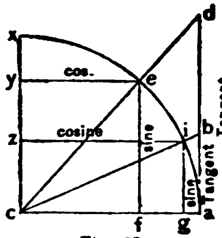


Fig. 68.

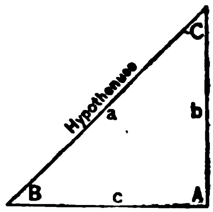


Fig. 69.

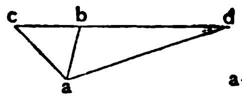


Fig. 70.

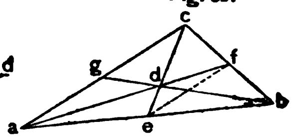


Fig. 71

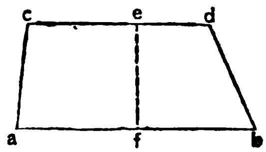


Fig. 72.

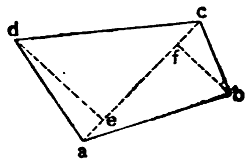


Fig. 73.

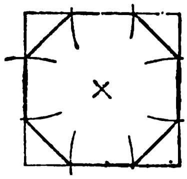


Fig. 74.

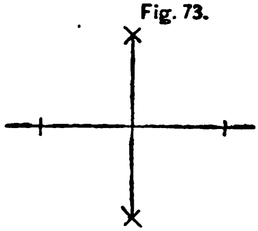


Fig 75.

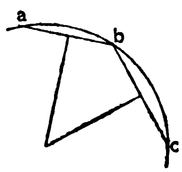


Fig. 76.

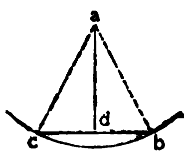


Fig. 77.

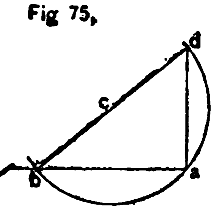


Fig. 78.

From a given point to let fall a perpendicular to a given line: From the given point, fig. 77, describe an arc cutting the line twice. With these two points proceed as in erecting a perpendicular at a given point, fig. 75, or bisect the portion of the line between the intersections, as at *d*, and draw the line *ad*, which is the perpendicular required.

To describe a circle passing through 3 given points: Join the points by 2 lines, as *ab* and *bc*, fig. 76, and construct a bisecting perpendicular on each. The perpendiculars intersect at the center of the required circle.

218. **Areas** are to each other as the squares of similar lines; similar triangles as the squares of corresponding sides, or of perpendiculars from corresponding angles to opposite sides, etc.

Squares are to each other as the squares of the sides or diagonals.

Other regular polygons are to each other as the squares of the sides or of the radii of inscribed or circumscribed circles.

Circles are to each other as the squares of diameters, or radii, or chords of equal arcs.

SPHERES AND CUBES.

219. The surface of a sphere $= 4\pi r^2 = 12.5664\ r^2 = 3.1416\ d^2 = 0.3183$ circ. squared $= 4\times$ area of a great circle $=$ diam. $\times$ circ. $=$ the curved surface of circumscribed cylinder.

The surfaces of two spheres are to each other as the squares of corresponding lines.

The volume of a sphere $= \frac{4}{3}\pi r^3 = 4.1888\ r^3 = 0.5236\ d^3 = 0.01689$ circ.$^3 = \frac{2}{3}$ diam. $\times$ area of great circle $= \frac{2}{3}$ vol. of circumscribed cylinder $= 0.5326$ vol. of circumscribed cube.

The volumes of spheres and cubes are to each other as the *cubes* of their corresponding *lines*, or the *squares* of corresponding *surfaces;* for a sphere, the radius, the diameter, the area of a great circle, or of any circle subtending equal angles at the center; for a cube, an edge, a diagonal of a side, or a diagonal of the cube. The diagonal of a cube $=$ the edge $\times$ 1.7321.

GRAVITATION.

220. The earth's attraction is measured by the increase in the velocity of a falling body which that attraction produces in a second of time. This quantity is represented by g, and its value at the surface of the earth on the equator is 32.092 feet. This means that if any body is falling freely its velocity at the end of any second of time is 32.092 feet per second greater than at the beginning of that second. If the body starts from rest, its velocity at the beginning is 0, and at the end of the first second is g. The velocity of any falling body at any instant equals g multiplied by the number of seconds the body has been falling. This relation is strictly true for a vacuum only, but for moderate heights is nearly correct in air.

The value of g varies slightly with the latitude as shown in the following table:

TABLE XVII.

221. **Values of g at** surface of earth in different latitudes:

Latitude.	Value of g in feet.	Latitude.	Value of g in feet.
Equator	32.09	52° 15′	32.19
20° 40′	32.11	60°	32.21
30°	32.13	69° 15′	32.23
37° 10′	32.15	90° pole	32.25
45°	32.17		

DO NOT REMOVE
OR
MUTILATE CARD

222. The value of **g** varies also with the distance from the center of the earth; or the distance above or below the surface, diminishing in both cases. This diminution is approximately 0.016 foot for each mile above the surface and 0.008 foot for each mile below.

223. The fundamental law of motion of falling bodies is $v^2 = 2gh$, in which v = the velocity at any point in feet per second; h, the distance through which the body has fallen from rest to the given instant. As $v = gt$, $h = \frac{1}{2}gt^2 = 16t^2$. These relations are strictly true only in vacuo, but for small, smooth, dense objects are approximately correct for motion in air up to 5 seconds.

CENTRIFUGAL FORCE.

224. If w = the weight of a revolving body and n the number of revolutions per minute, r = the radius of revolution or the distance of center of gravity of the body from center of motion, and c = the centrifugal force or pull on the radius in pounds, then $c = 0.00034wrn^2$

Equivalents of measure.

LENGTHS.

1 meter, m = 10 decimeters, dm = 100 centimeters, cm = 1000 millimeters, mm.
1 meter, m = 0.1 decameter, dkm = 0.01 hectometer, hm = 0.001 kilometer, km.
1 meter, m = 39.37 inches, U. S. Standard = 39.370113 inches, British standard.
1 millimeter, mm = 1000 microns, μ = 0.03937 inch = 39.37 mils.

Meters, m.	Inches, in.	Feet, ft.	Yard, yd.	Rods, r.	Chains, ch.	Miles, U. S. Statute.	Miles, U. S. Nautical.	Kilometers, km.
1	39.37	3.28083	1.09361	0.19884	0.04971	$0.\overset{3}{0}6214$	$0.\overset{3}{0}5396$	0.001
0.02540	1	0.08333	0.02778	$0.\overset{2}{0}5051$	$0.\overset{3}{0}1263$	$0.\overset{4}{0}1578$	$0.\overset{4}{0}1371$	$0.\overset{4}{0}2540$
0.30480	12	1	0.33333	0.06061	0.01515	$0.\overset{3}{0}1894$	$0.\overset{3}{0}1645$	$0.\overset{3}{0}3048$
0.91440	36	3	1	0.18182	0.04545	$0.\overset{3}{0}5682$	$0.\overset{3}{0}4934$	$0.\overset{3}{0}9144$
5.02921	198	16.5	5.5	1	0.25	$0.\overset{2}{0}3125$	$0.\overset{2}{0}2714$	$0.\overset{2}{0}5029$
20.1168	792	66	22	4	1	0.01250	0.01085	0.02012
1609.35	63360	5280	1760	320	80	1	0.86839	1.60935
1853.25	72962.5	6080.20	2026.73	368.497	92.1243	1.15155	1	1.85325
1000	39370	3280.83	1093.61	198.838	49.7096	0.62137	0.53959	1

1 yard, U. S. = 1.0000029 yards British. 1 yard British = 0.9999971 yard U. S.
1 chain, Gunter's = 100 links. 1 link = 7.92 inches.
1 cable length, U. S. = 120 fathoms = 960 spans = 720 feet = 219.457 meters.
1 league, U. S. = 3 statute miles = 24 furlongs.
1 international geographical mile = $\frac{1}{15}$° at equator = 7422 m = 4.611808 U. S. statute miles.
1 international nautical mile = $\frac{1}{60}$° at meridian = 1852 m = 0.999326 U. S. nautical miles.
1 U. S. nautical mile = $\frac{1}{60}$° of circumference of sphere whose surface equals that of the earth = 6080.27 feet = 1.15155 statute miles = 1853.27 meters.
1 British nautical mile = 6080.00 feet = 1.15152 statute miles = 1853.19 meters.

SURFACES AND AREAS.

1 sq. meter, m^2 = 100 sq. decimeters, dm^2 = 10000 sq. centimeters, $cm.^2$
1 sq. meter, m^2 = 0.01 are, a = 0.0001 hectare, ha.
1 sq. millimeter, mm^2 = 0.01 cm^2 = 0.00155 sq. inch = 1217.36 circ. mils.
1 are, a = 1 sq. decameter, dkm = 0.0247104 acre.

Square meters, m^2.	Square inches, sq. in.	Square feet, sq. ft.	Square yards, sq. yd.	Square rods, sq. r.	Acres, A.	Hectares, ha.	Square miles, statute.	Square kilometers, km^2.
1	1550.00	10.7639	1.19599	0.03954	$0.\overset{3}{0}2471$	0.00001	$0.\overset{6}{0}3861$	$0.\overset{5}{0}1$
$0.\overset{3}{0}6452$	1	$0.\overset{2}{0}6944$	$0.\overset{3}{0}7716$	$0.\overset{4}{0}2551$	$0.\overset{6}{0}1594$	$0.\overset{7}{0}6452$	$0.\overset{9}{0}2491$	$0.\overset{9}{0}6452$
0.09290	144	1	0.11111	$0.\overset{2}{0}3673$	$0.\overset{4}{0}2296$	$0.\overset{5}{0}9290$	$0.\overset{7}{0}3587$	$0.\overset{7}{0}9290$
0.83613	1296	9	1	0.03306	$0.\overset{3}{0}2066$	$0.\overset{4}{0}8361$	$0.\overset{6}{0}3228$	$0.\overset{6}{0}8361$
25.2930	39204	272.25	30.25	1	0.00625	$0.\overset{2}{0}2529$	$0.\overset{5}{0}9766$	$0.\overset{4}{0}2529$
4046.87	6272640	43560	4840	160	1	0.40469	$0.\overset{2}{0}1563$	$0.\overset{2}{0}4047$
10000	15499969	107639	11959.9	395.366	2.47104	1	$0.\overset{2}{0}3861$	0.01
2589999		27878400	3097600	102400	640	259.000	1	2.59000
1000000		10763867	1195985	39536.6	247.104	100	0.38610	1

1 sq. rod, sq. pole, or sq. perch = 625 sq. links = $\frac{1}{160}$ acre.
1 sq. chain, Gunter's = 16 sq. rods = $\frac{1}{10}$ acre.
1 acre = 4 sq. roods = 160 sq. rods. Square of 1 acre = 208.7103 feet square.
Notations $\overset{2}{0}$, $\overset{3}{0}$, $\overset{4}{0}$, etc., indicate that the $\overset{2}{0}$, $\overset{3}{0}$, $\overset{4}{0}$, etc., are to be replaced by 2, 3, 4, etc., ciphers.
Example: 1 sq. rod = $0.\overset{5}{0}9766$ = 0.000009766 sq. miles.